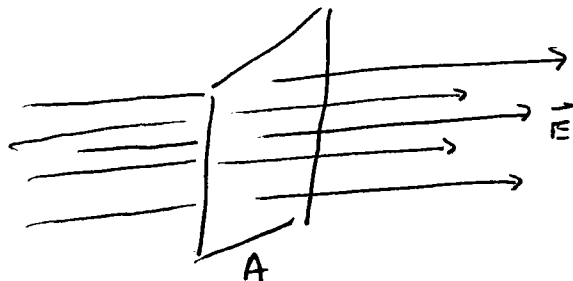


Electric Flux Thru a surface

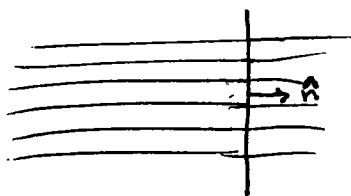
$\equiv \Phi \equiv$ # of \vec{E} field lines crossing the surface.



$\vec{E} \perp$ Surface A

$$\Phi \equiv |\vec{E}| A$$

Use Transparency



vs.



$$d\Phi = |\vec{E}| dA$$

$$\vec{E} \cdot \hat{n} = |\vec{E}|$$

A effectively becomes

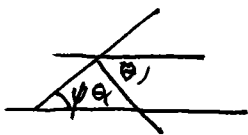
$$(A \sin \theta) E$$

height goes from A to $A \sin \theta$

$$\vec{n} \cdot \vec{E} = |\vec{E}| \cos \theta$$



$$\hat{n} \cdot \vec{E} = \hat{n} \cos \theta$$



$$\sin \psi = \cos \theta$$

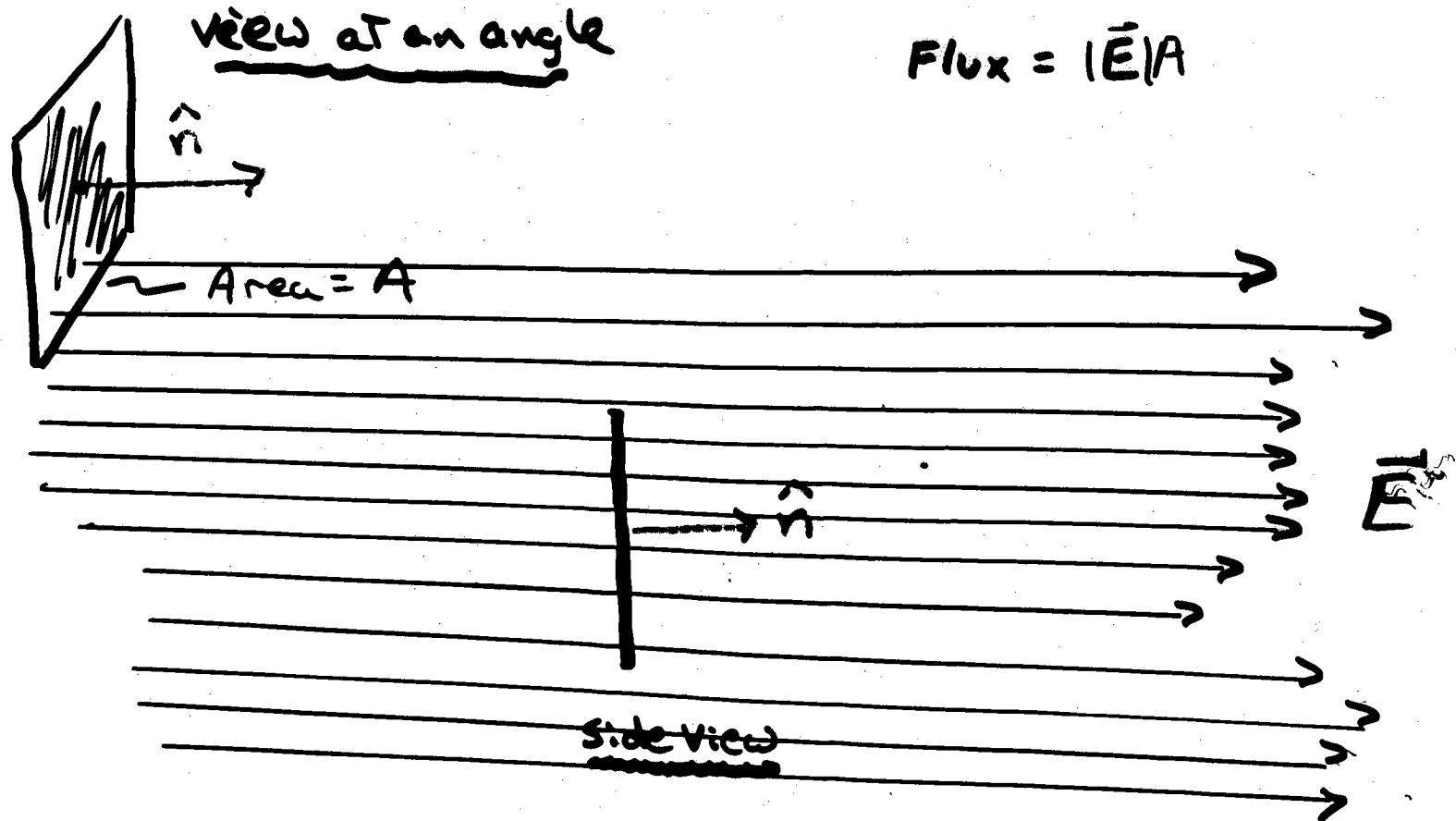
$$d\Phi = |\vec{E}| dA \cos \theta$$

$$d\Phi = \vec{E} \cdot d\vec{A} = E dA \cos \theta = \vec{E} \cdot \hat{n} dA$$

13-792 500 SHEETS, FULLER, 5 SQUARE
42-381 50 SHEETS, EYE-EASE, 5 SQUARE
42-382 100 SHEETS, EYE-EASE, 5 SQUARE
42-383 100 SHEETS, EYE-EASE, 5 SQUARE
42-384 100 RECYCLED, WHITE, 5 SQUARE
42-385 200 RECYCLED, WHITE, 5 SQUARE
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$$\vec{E} \cdot \hat{n} = |\vec{E}| |\hat{n}| \cos \theta = |\vec{E}|$$

$$\theta = 0$$

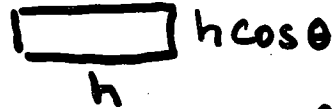
$$|\hat{n}| = 1$$

\vec{E} view



$$A = h^2$$

\Rightarrow



$$A = h h \cos \theta$$

effective

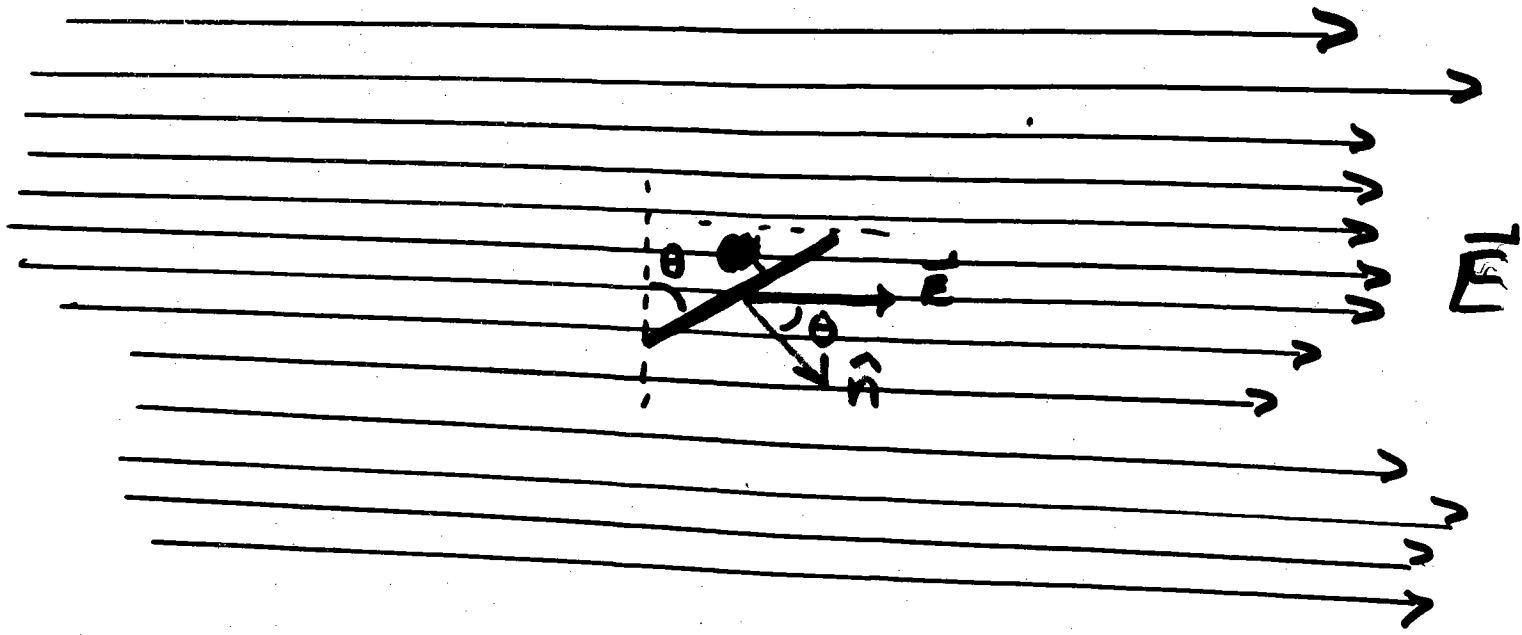
$$\text{Flux} = |\vec{E}| A_{\text{effective}} = E A \cos \theta$$

General Expression

$$\text{Flux} = \vec{E} \cdot \hat{n} A$$

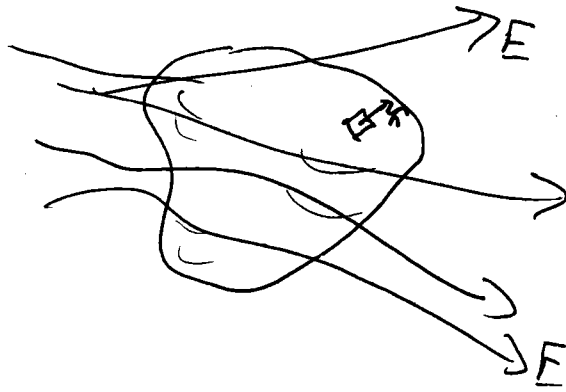
or

$$d\phi = \vec{E} \cdot \hat{n} dA$$



$$\vec{E} \cdot \hat{n} = |\vec{E}| |\hat{n}| \cos \theta$$

Arbitrary surface

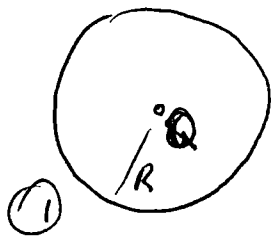


⇒ What is sign of $\vec{E} \cdot \vec{n}$

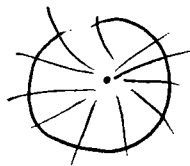
$$\text{Flux} \equiv \Phi = \int_{\text{Surface}} \vec{E} \cdot \vec{n} \, dA$$

$$\Phi = \int_S \vec{E} \cdot d\vec{A}$$

Gauss's Law



consider spherical surface surrounding pt charge Q at origin



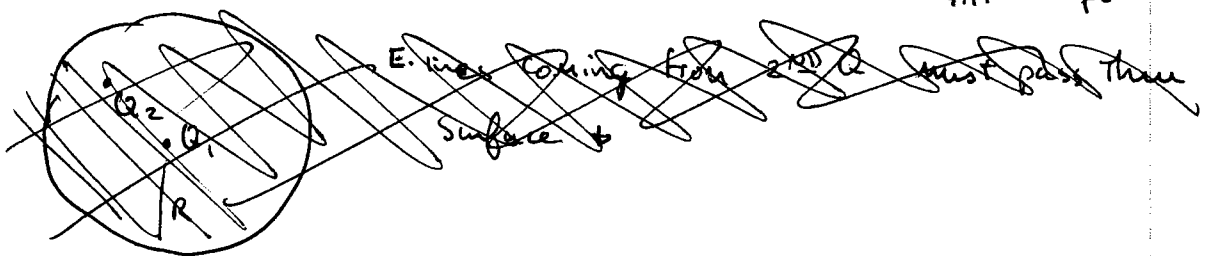
$$\Phi = \int \vec{E} \cdot d\vec{A} = |\vec{E}| \int dA = |\vec{E}| 4\pi R^2$$

$$= |\vec{E}| A = \frac{kQ}{r^2} A$$

TOTAL flux $\propto Q$

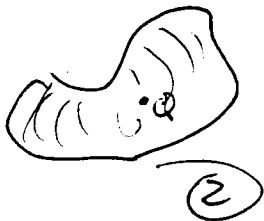
$$\Phi = kQ 4\pi = \frac{Q}{\epsilon_0}$$

$$\text{Also } \vec{E} = \frac{kQ 4\pi}{A} = \frac{kQ 4\pi}{4\pi r^2} = \frac{kQ}{r^2} \vec{n}$$



consider a different surface

↪ The total # of lines is the same

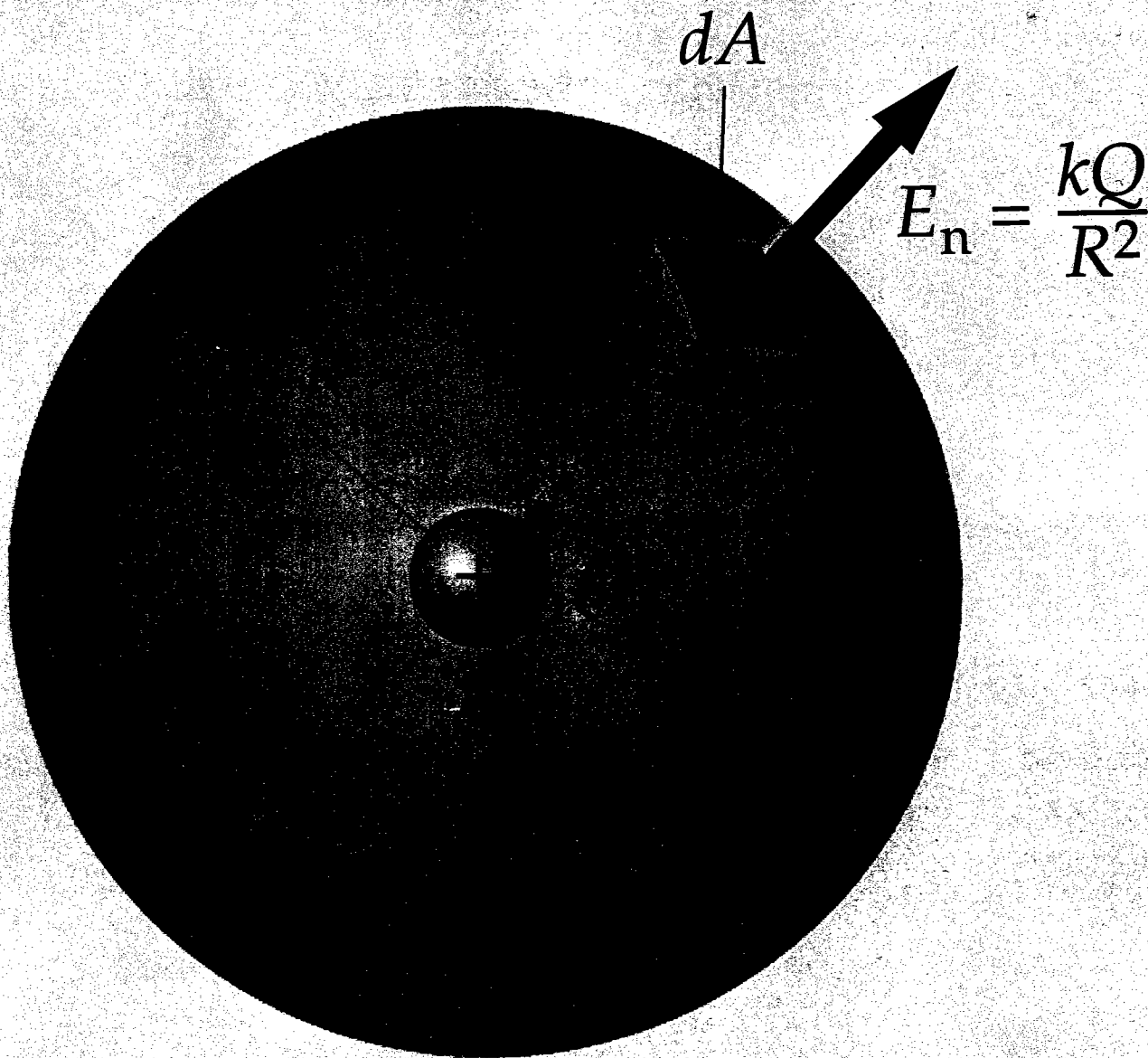


$$\therefore \Phi_{(1)} = \Phi_{(2)}$$

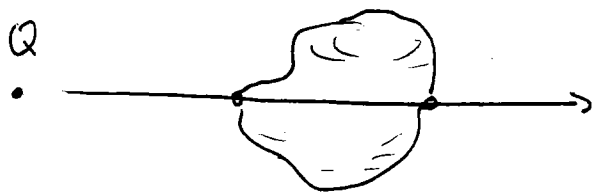
$$\therefore \int \vec{E} \cdot d\vec{A} = kQ 4\pi_{\text{inside}}$$

Transparency 6
Figure 23-15, page 698
Spherical surface enclosing a point charge

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Consider Q outside



A line of force entering
also leaves

Gauss's Law

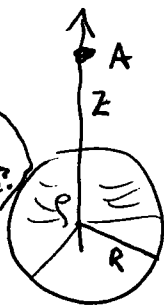
$$\int_{\text{surf}} \vec{E} \cdot d\vec{A} = \frac{1}{4\pi\epsilon_0} 4\pi Q_{\text{inside}} = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

Also one of Maxwell's equations

Why do you care?

One can use Gauss's law + Symmetry to solve for \vec{E}
Almost Trivially

What is symmetry?
What Gaussian surface do I choose?
 $\Rightarrow \infty$ line charge
 $\Rightarrow \infty$ plane
 \Rightarrow sphere



recall our awful example of Field outside
Sphere of uniform charge density

consider $r > R$

$$\Phi = \frac{Q_{\text{inside}}}{\epsilon_0} = \int \vec{E} \cdot d\vec{A} = |\vec{E}| \int dA = |\vec{E}| 4\pi r^2$$

$$|\vec{E}| = \frac{Q_{\text{inside}}}{4\pi\epsilon_0 r^2}$$

same as pt charge w/ $q = Q_{\text{inside}}$

$$Q_{\text{inside}} = \frac{4}{3}\pi R^3 \rho$$

$$|\vec{E}|_{r > R} = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi\epsilon_0 r^2} = \frac{R^3 \rho}{3\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

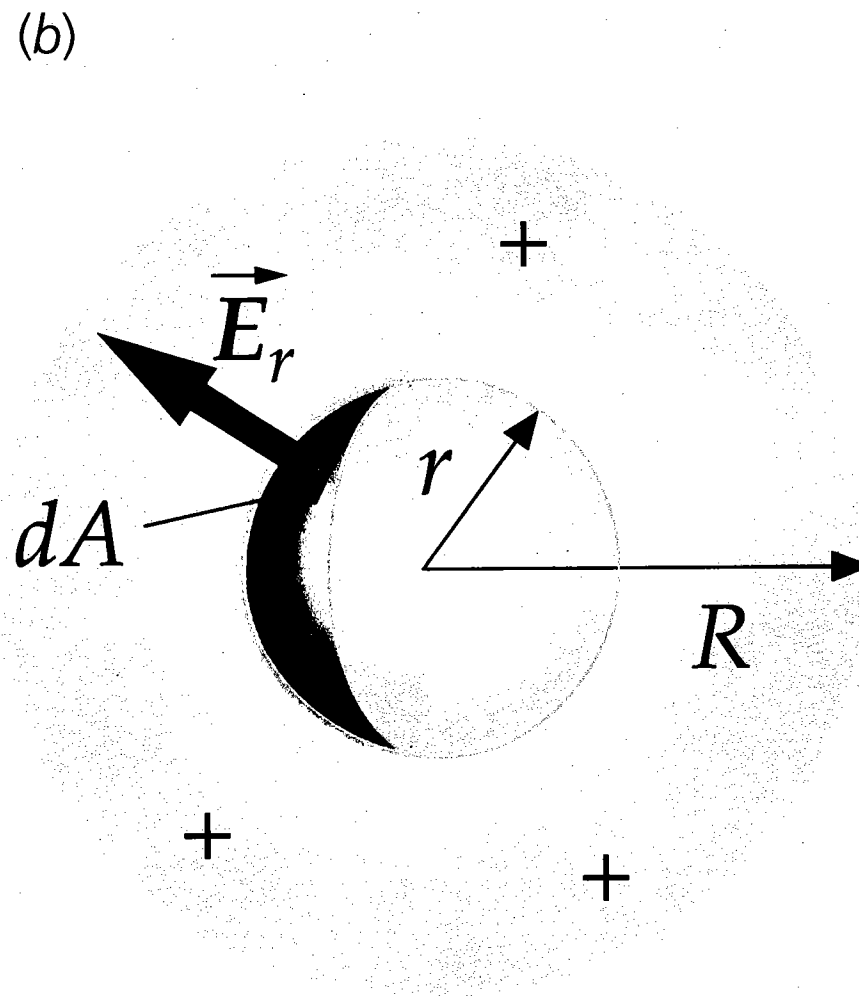
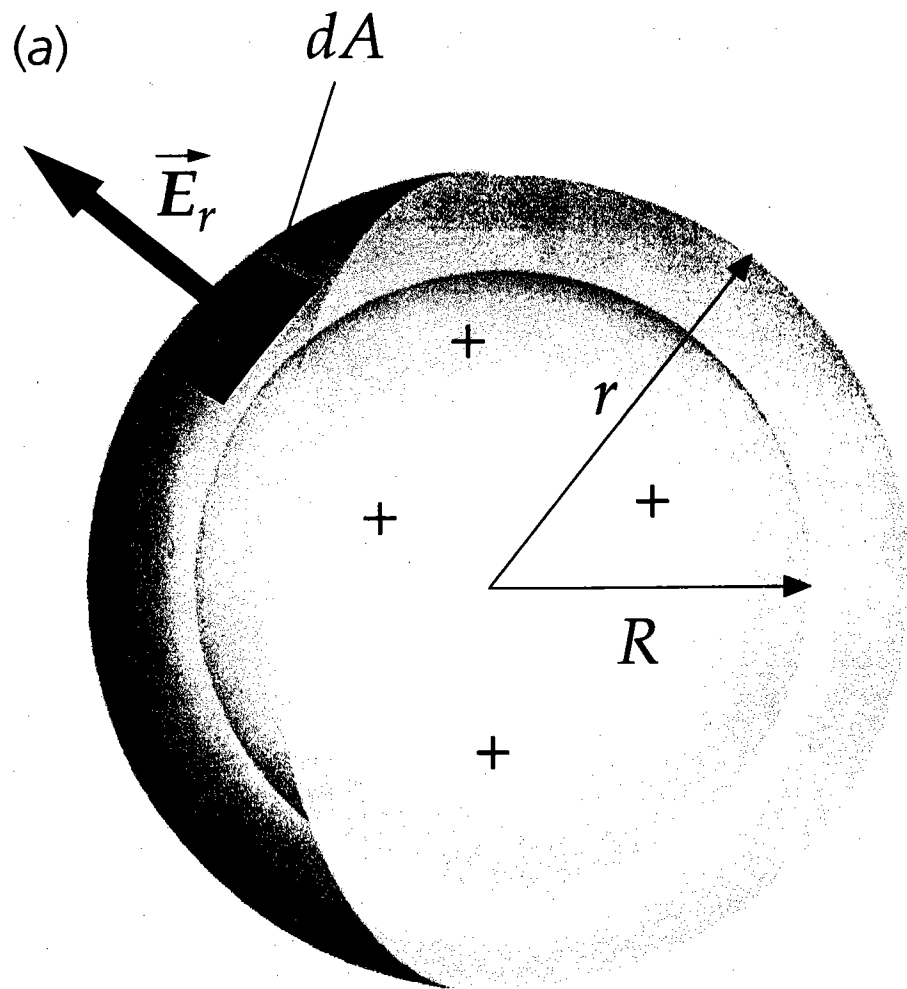
by symmetry E is in \hat{r} direction

Transparency 8

Figure 23-23, page 704

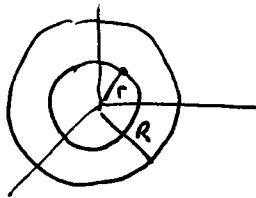
Gaussian surface outside a spherical shell (left) and inside a spherical shell (right)

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$$\text{So } \vec{E} \text{ for } r > R = \frac{\rho R^3}{3\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad r \geq R$$

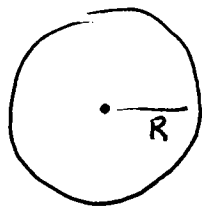
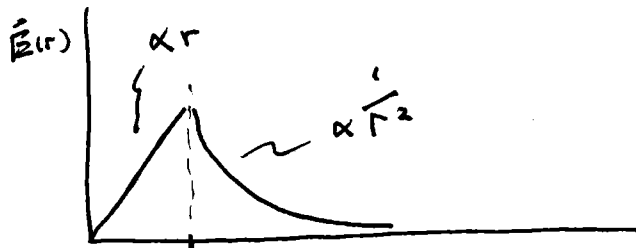
Suppose $r \leq R$



$$|\vec{E}| = \frac{Q_{\text{inside}}}{4\pi\epsilon_0 r^2}$$

$$Q_{\text{inside}} = \rho \frac{4}{3} \pi r^3$$

$$\vec{E} = \frac{\rho \frac{4}{3} \pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho r}{3\epsilon_0} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r^2 \quad r \leq R$$



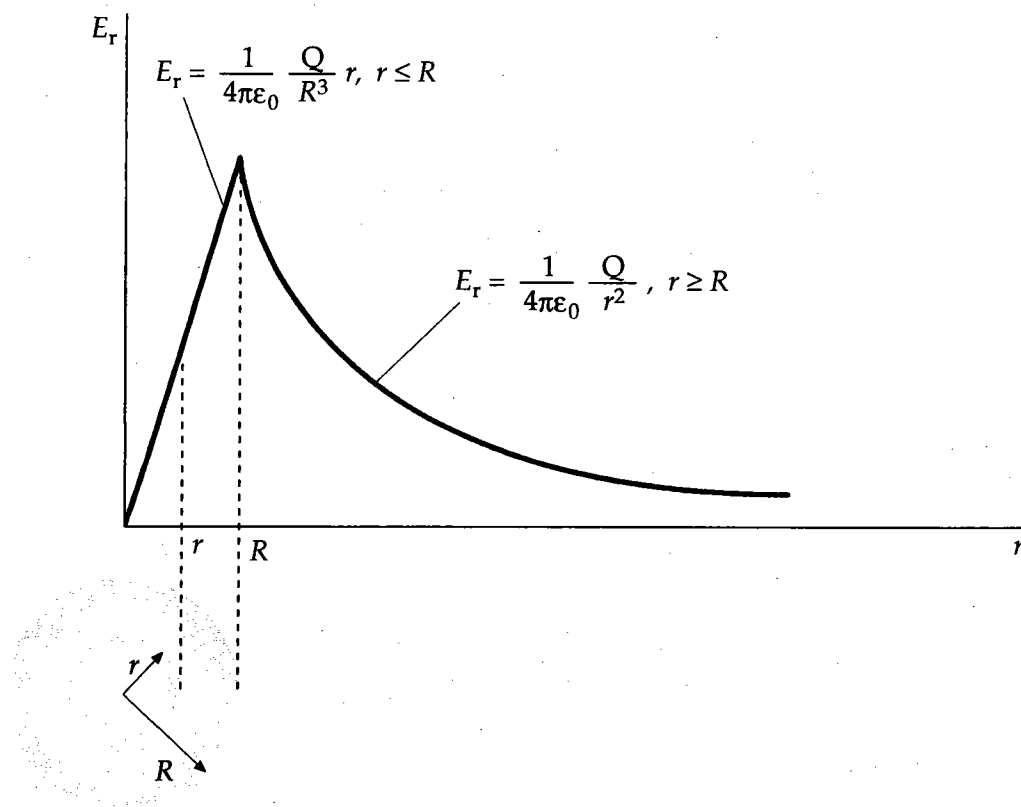
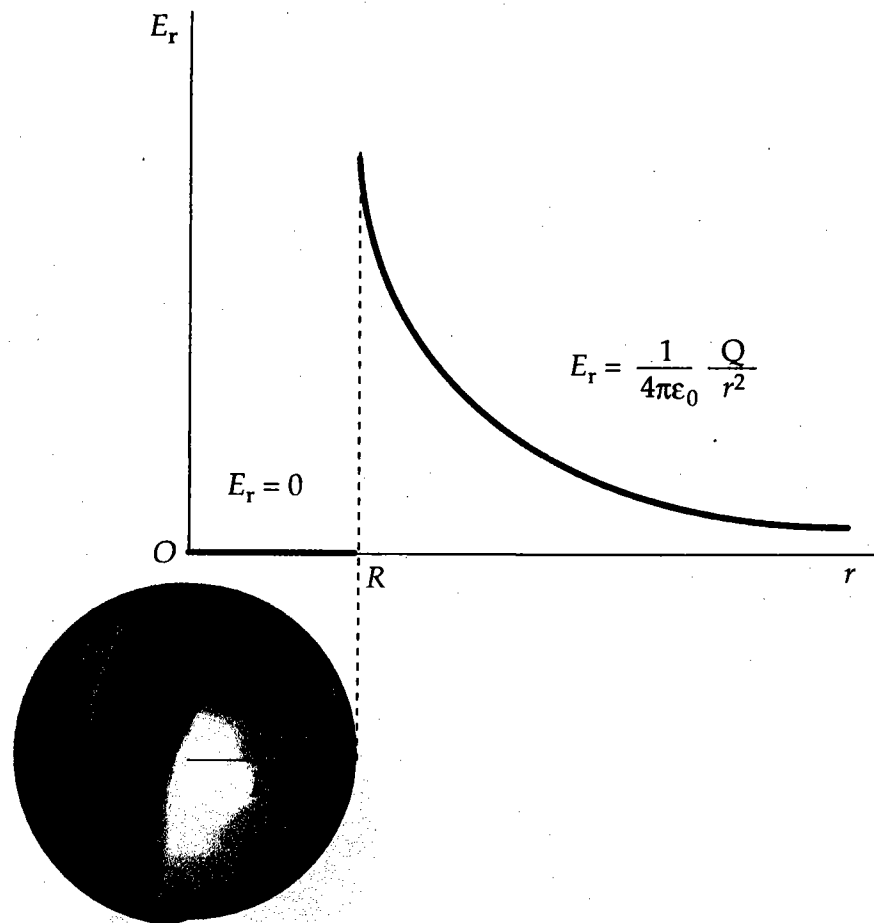
do Telegram

Transparency 10

Figure 23-21, page 702; Figure 23-24, page 705

E due to a spherical shell of charge (left) and due to a solid sphere of charge (right)

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P114 Telegram:

\vec{E} can be calculated directly from ρ (or λ , or Q .)

$$\vec{E} = \int \frac{k \rho}{r^2} \hat{r}$$

often This is hard

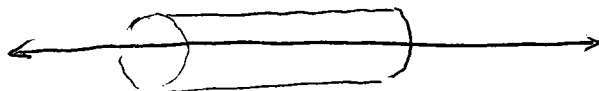
Use Gauss's Law when symmetry allows
 $|\vec{E}|$ to be Moved out of integral

$$\int \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0} \rightarrow |\vec{E}| \int dA = \frac{Q}{\epsilon_0}$$

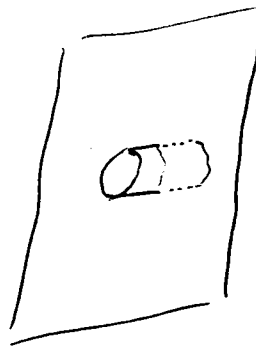
Symmetries often seen



spherical
spherical
surface



line \rightarrow cylindrical
surface



Plane - pillbox
cylindrical
surface

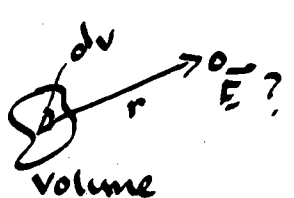
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100% RECYCLED WHITE: 8 SQUARE
Made in U.S.A.

\vec{E} calculated directly from charge distribution

$$\vec{E} = \int \frac{k dq \hat{r}}{r^2}$$

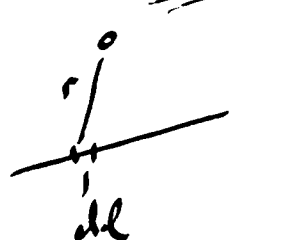
Volume



A diagram showing a small volume element dV within a larger volume. A vector \vec{r} points from the volume element to a point labeled $\vec{E}?$. The word "Volume" is written below the diagram.

$$\vec{E} = \int \frac{k \rho dV \hat{r}}{r^2}$$

line



A diagram showing a small line element dl on a line. A vector \vec{r} points from the line element to a point. The word "line" is written above the diagram.

$$\vec{E} = \int \frac{k \lambda dl \hat{r}}{r^2}$$

Surface ... etc

This is hard !!

Gauss's law allows one to calculate \vec{E} in a much easier way if symmetry allows and you choose the right Gaussian surface

$$\int_{\text{Gaussian Surf}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

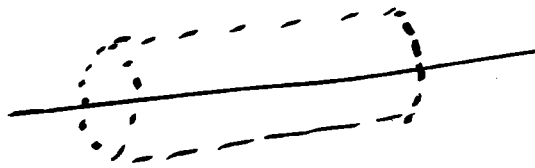
Look at symmetry of charge distribution and
choose Gaussian surface so that
 $\vec{E} \cdot d\vec{A}$ is easy to evaluate

Examples

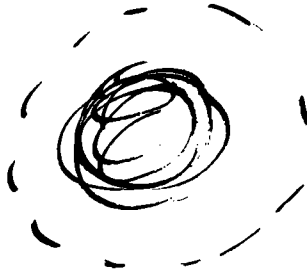
Point



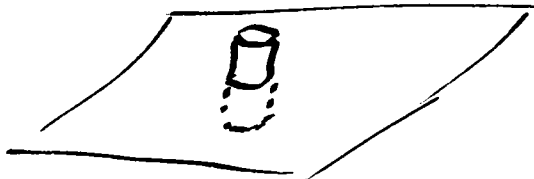
Line



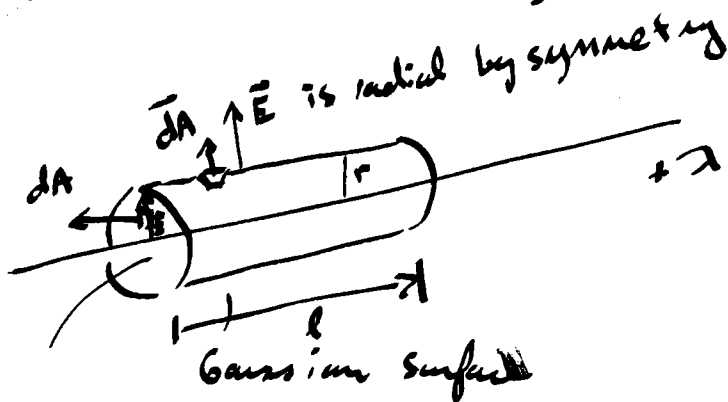
sphere



Plane



What is \vec{E} for ∞ line charge



$$\int \vec{E} \cdot d\vec{A} = \int_{\text{endcap 1}} \vec{E} \cdot d\vec{A} + \int_{\text{endcap 2}} \vec{E} \cdot d\vec{A} + \int_{\text{cylinder}} \vec{E} \cdot d\vec{A}$$

$\vec{E} \cdot d\vec{A} = 0$ \vec{E} is constant at a fixed radius
 $\vec{E} \perp \text{to } d\vec{A}$

$$\int \vec{E} \cdot d\vec{A} = \int_{\text{cylinder}} E \, dA = E \, 2\pi r l = \frac{Q_{enc}}{\epsilon_0}$$

by Gauss's law

$$Q_{enc} = \lambda l$$

$$\therefore E = \frac{\lambda l}{2\pi r l \epsilon_0} = \frac{\lambda}{2\pi r \epsilon_0} \quad \text{radially outward}$$