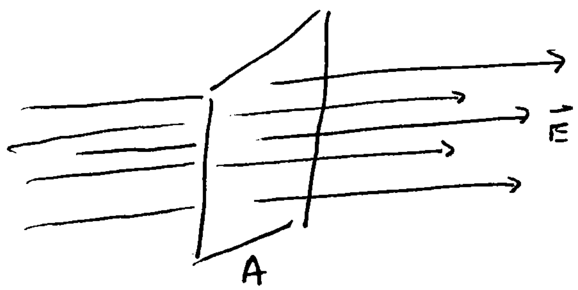
Electric Flux Thru a surface

$= \phi \equiv$ # of $\vec{E}$ field lines crossing the surface.

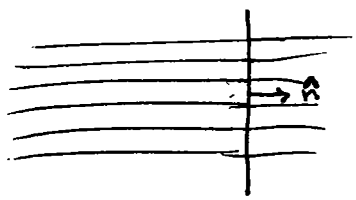


$\vec{E} \perp$ ~~A~~ surface A

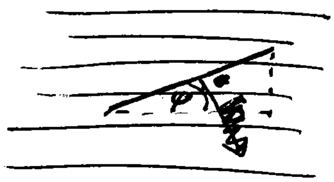
$\phi \equiv |\vec{E}|\, A$

Use Transparency

Presentation Better w/ Transparencies



vs.



$d\phi = |\vec{E}| dA$

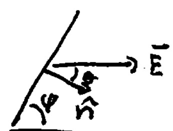
$\vec{E} \cdot \hat{n} = |\vec{E}|$

A effectively becomes

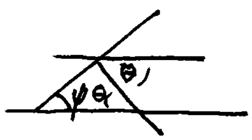
$(A \sin \varphi)\, E$

height goes from A To $A \sin\theta$

$| \rightarrow \hat{n} \cdot \vec{E} = \vec{E}$



$\hat{n} \cdot \vec{E} = \hat{n} \cos\theta$

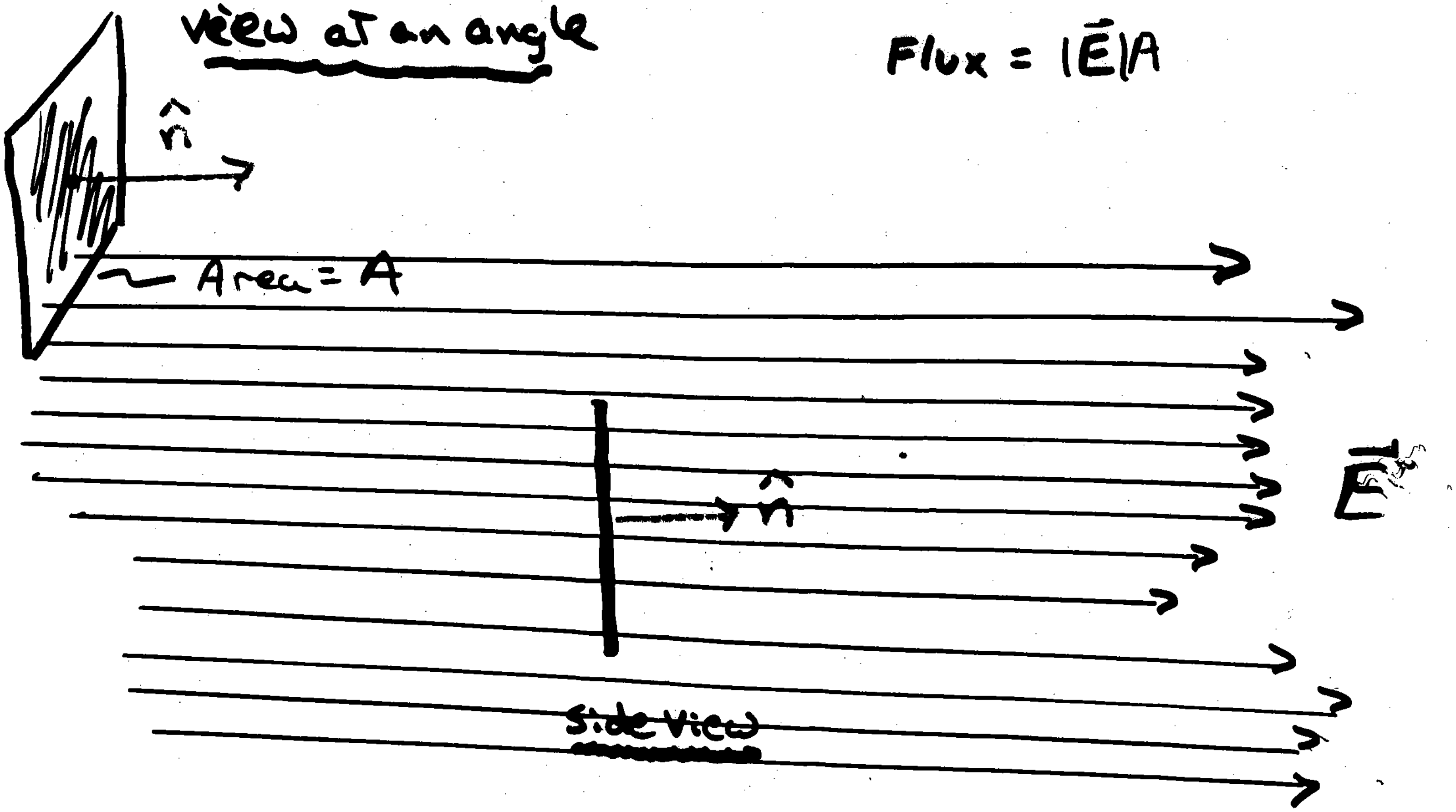


$\sin \varphi = \cos \theta$

$d\phi = |\vec{E}| dA \cos\theta$

$d\phi = \vec{E} \cdot d\vec{A} = \cancel{E\,dA}\quad \vec{E} \cdot \hat{n}\, dA$

$\text{Flux} = |\vec{E}|A$

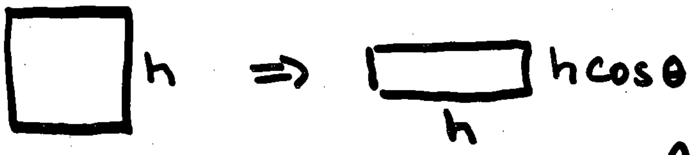


$$\vec{E}\cdot\hat{n} = |\vec{E}||\hat{n}|\cos\theta = |\vec{E}|$$

$$\theta = 0$$

$$|\hat{n}| = 1$$

$\vec{E}$ view



$A = hh$

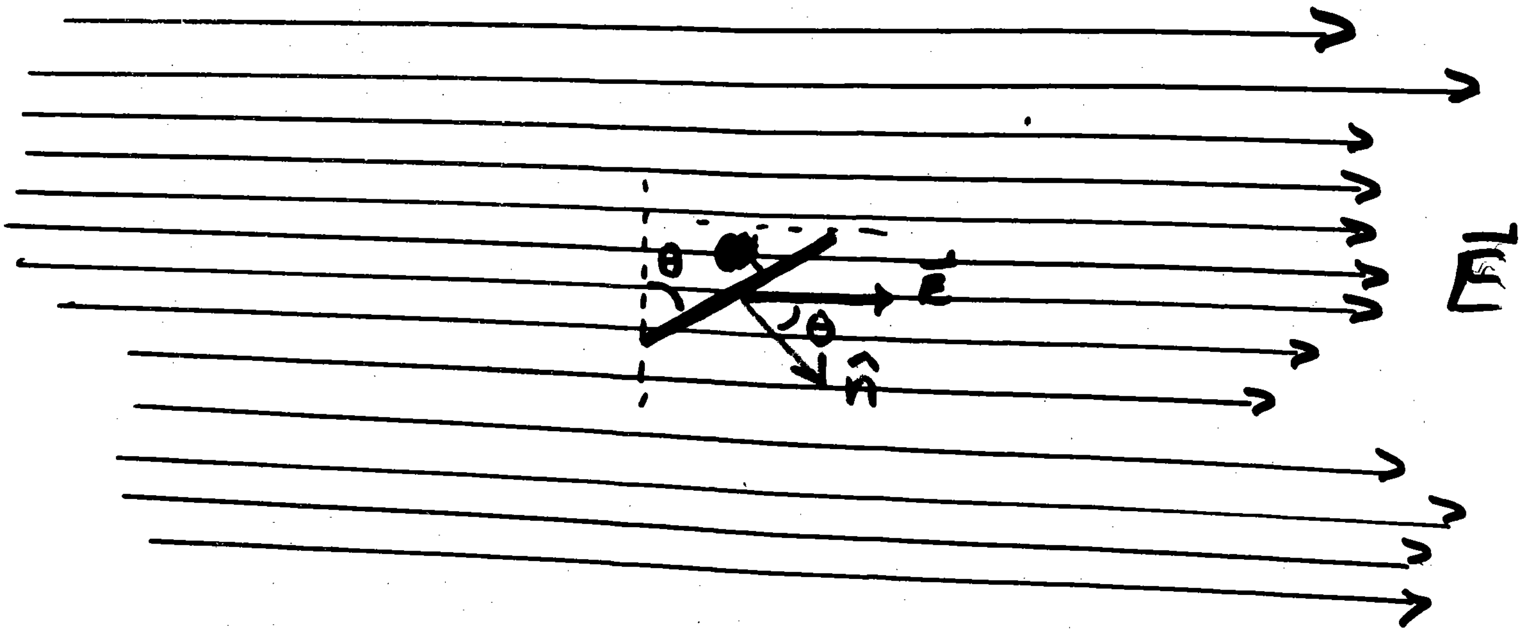
$A_{effective} = hh\cos\theta$

Flux $= |\vec{E}|\, A_{effective} = EA\cos\theta$

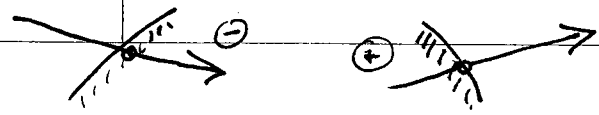
General Expression

Flux $= \vec{E}\cdot\hat{n}\, A$ or

$d\phi = \vec{E}\cdot\hat{n}\, d$

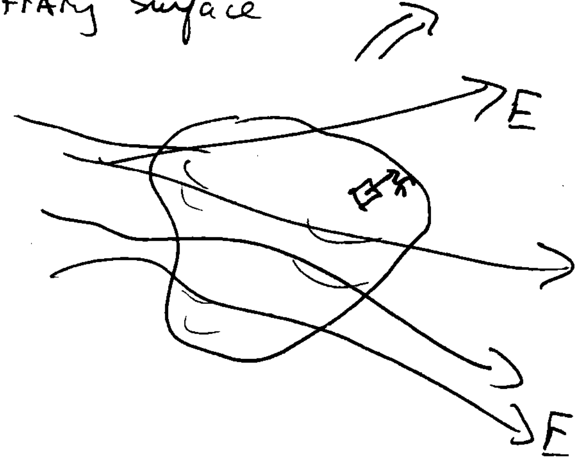


$\vec{E}\cdot\hat{n} = |\vec{E}||\hat{n}|\cos\theta$



$\Rightarrow$ what is sign of $\vec{E}\cdot\hat{n}$

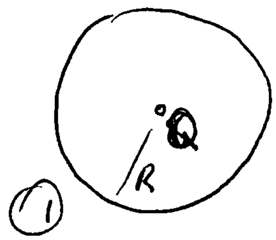
Arbitrary Surface



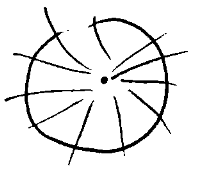
flux $\equiv \phi = \int_{\text{Surface}} \vec{E}\cdot\hat{n}\, dA$

$\phi = \int_S \vec{E}\cdot d\vec{A}$

Gauss's Law



consider spherical surface surrounding pt charge Q at origin



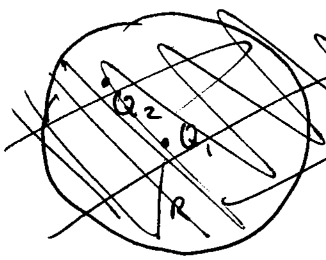
$\phi = \int \vec{E}\cdot d\vec{A} = |\vec{E}|\int dA = |\vec{E}|\, 4\pi R^2$

$= |\vec{E}|\, A = \frac{kQ}{r^2} A$

$\phi = kQ\, 4\pi = \frac{Q}{\epsilon_0}$

Total flux $\propto Q$

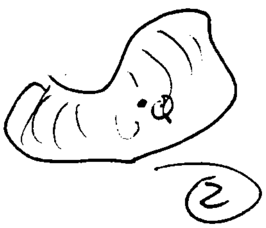
Also $\vec{E} = \frac{kQ 4\pi}{A} = \frac{kQ 4\pi}{4\pi r^2} = \frac{kQ}{r^2}\hat{r}$



~~E. lines coming from 2nd Q must pass thru surface~~

consider a different surface $\rightsquigarrow$ The total # of lines is the same

$\therefore \quad \phi_{①} = \phi_{②}$



$\therefore \int \vec{E}\cdot d\vec{A} = kQ_{\text{inside}} 4\pi$

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Transparency 6
Figure 23-15, page 698
Spherical surface enclosing a point charge

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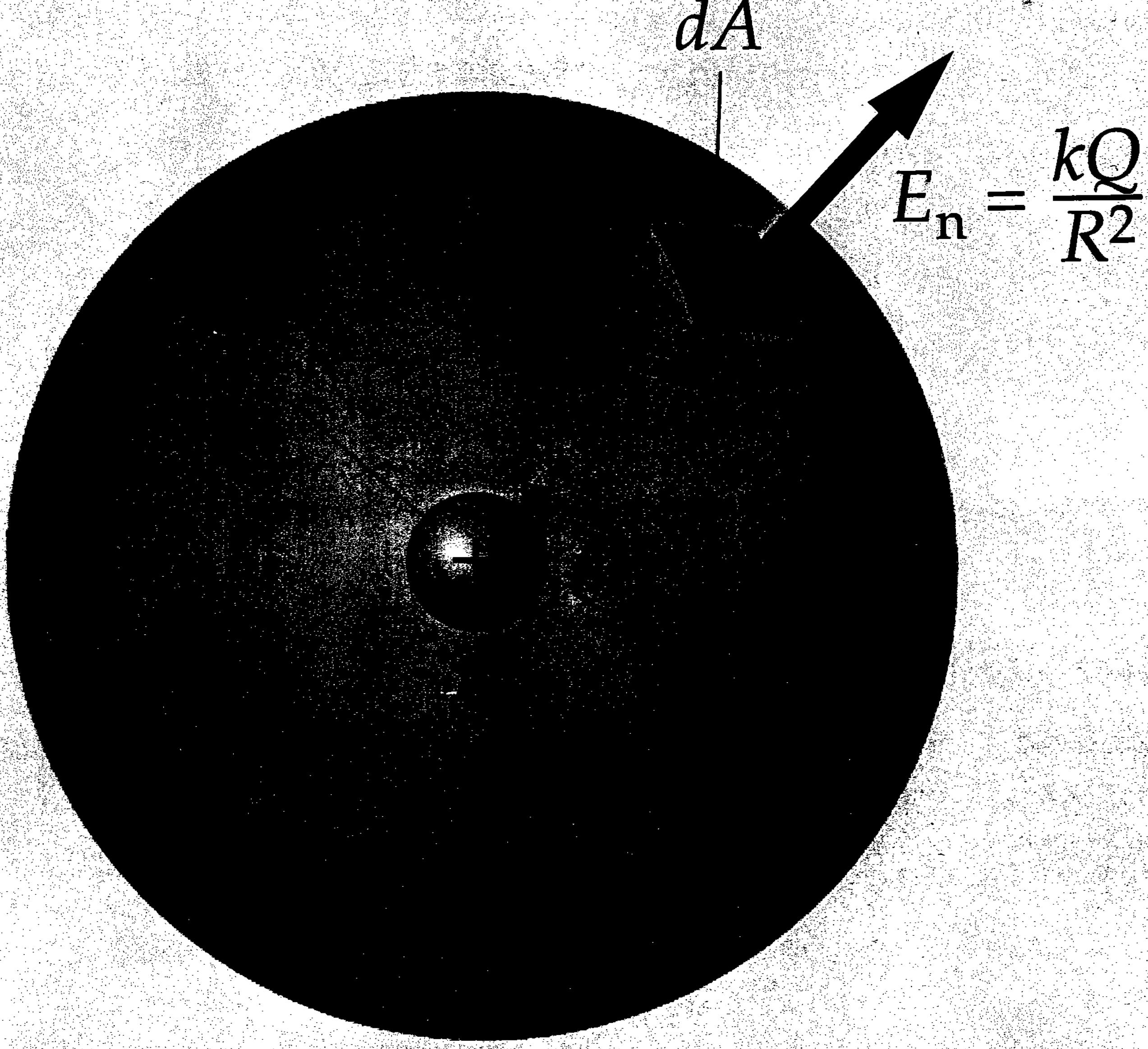
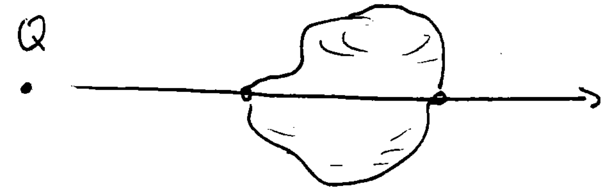


Consider Q outside



A line of force entering also leaves

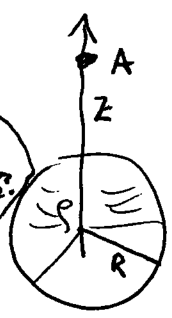
Gauss's Law

$$\int_{surf} \vec{E}\cdot d\vec{A} = \frac{1}{4\pi\epsilon_0} 4\pi Q_{inside} = \frac{1}{\epsilon_0} Q_{inside}$$

Also one of Maxwell's equations

Why do you care?

One can use Gauss's law + Symmetry to solve for $\vec{E}$ ALMOST Trivially



what is symmetry?
What Gaussian surface do I choose?
⇒ ∞ line charge
⇒ ∞ plane
⇒ sphere

recall our awful example of Field outside Sphere of uniform charge density

Consider $r > R$

$$\phi = \frac{Q_{inside}}{\epsilon_0} = \int \vec{E}\cdot d\vec{A} = |\vec{E}| \int dA = |\vec{E}|\, 4\pi r^2$$

$$|\vec{E}| = \frac{Q_{inside}}{4\pi\epsilon_0 r^2}$$ same as pt charge w/ $q = Q_{inside}$

$$Q_{inside} = \frac{4}{3}\pi R^3 \rho$$

$$|\vec{E}|_{r>R} = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi\epsilon_0 r^2} = \frac{R^3\rho}{3\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

by symmetry E is in $\hat{r}$ direction

Transparency 8
Figure 23-23, page 704
Gaussian surface outside a spherical shell (left) and inside a spherical shell (right)

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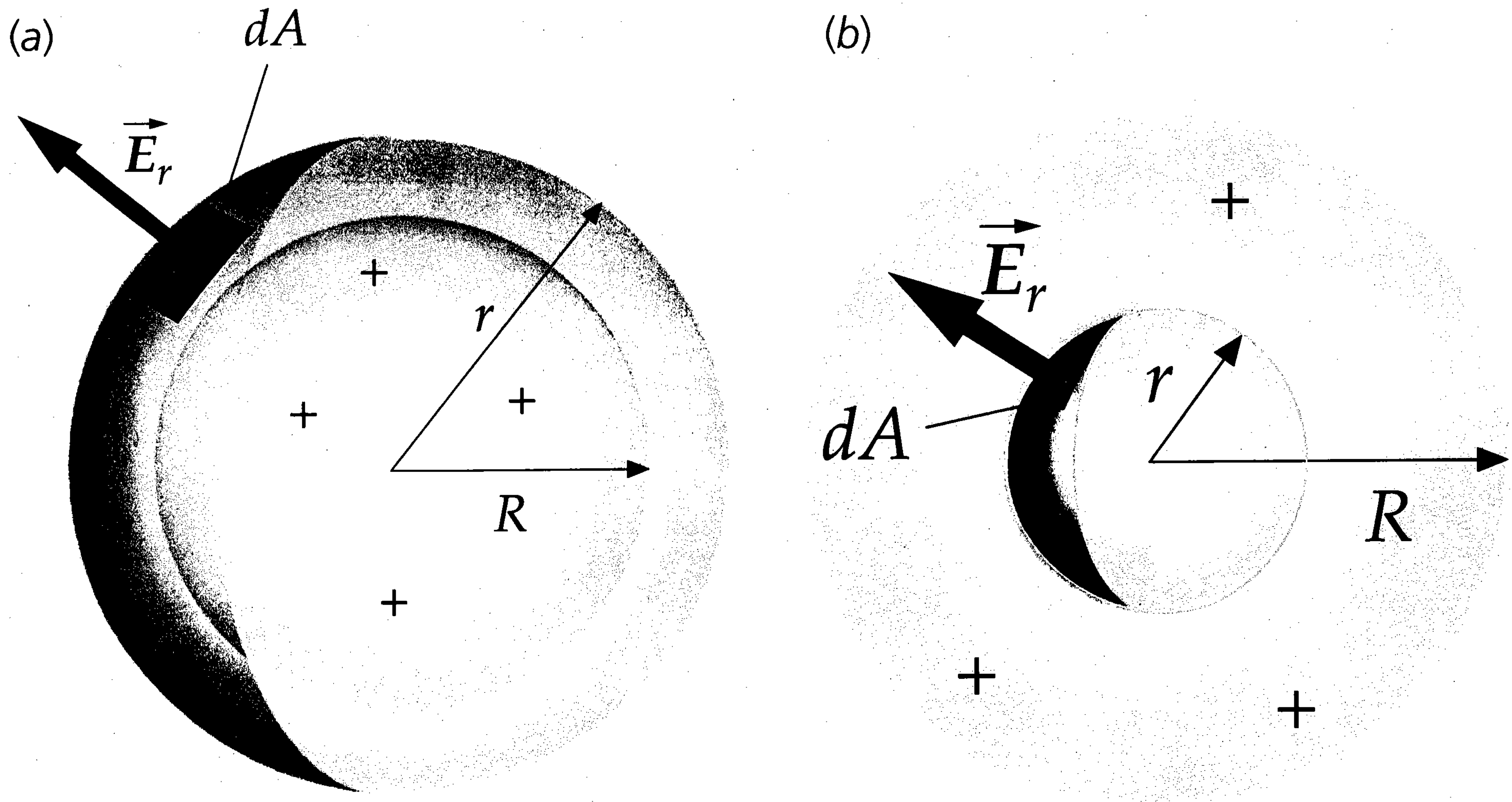
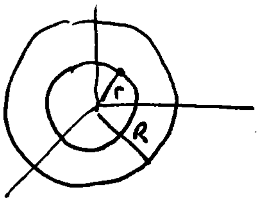


50

$$\vec{E}_{r>R} = \frac{\rho R^3}{3\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad r \geq R$$

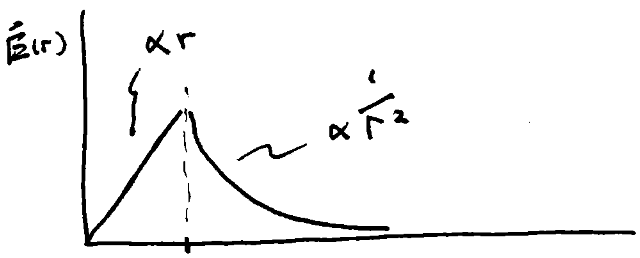
Suppose $r \leq R$

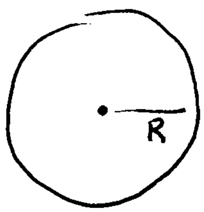


$$|\vec{E}| = \frac{Q_{inside}}{4\pi\epsilon_0 r^2}$$

$$Q_{inside} = \rho \tfrac{4}{3}\pi r^3$$

$$\vec{E} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho r}{3\epsilon_0} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\hat{r} \qquad r \leq R$$





do Telegram

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Transparency 10
Figure 23-21, page 702; Figure 23-24, page 705
E due to a spherical shell of charge (left) and due to a solid sphere of charge (right)

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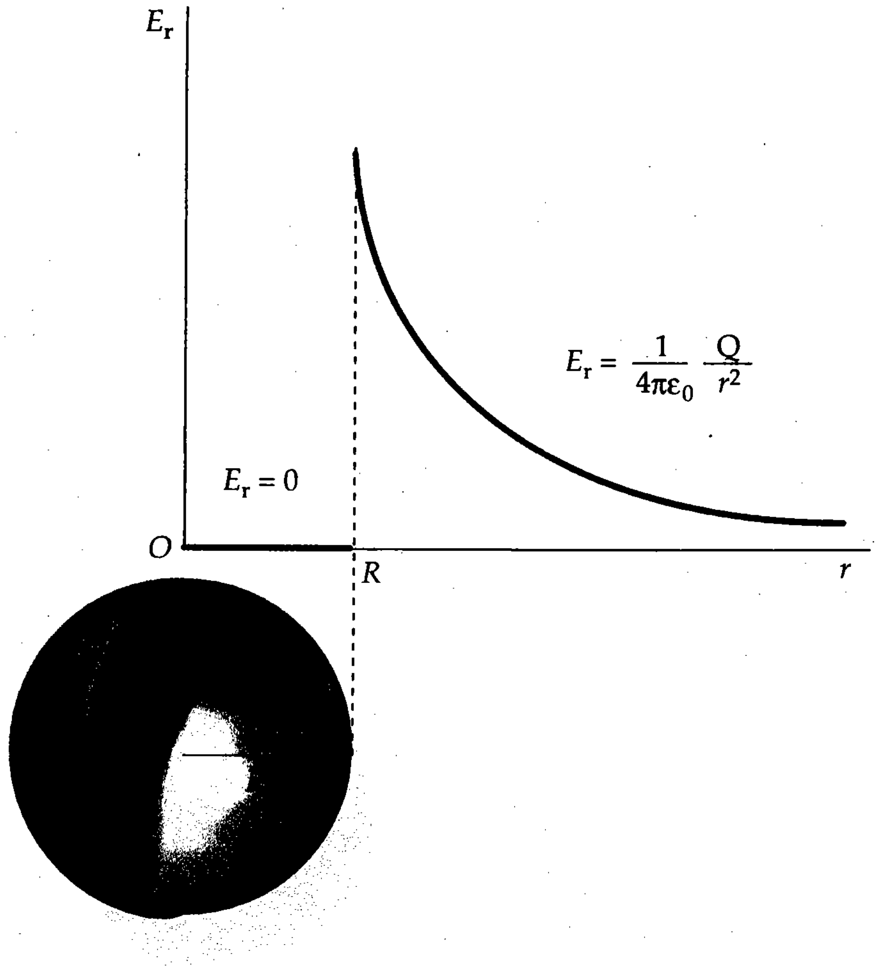
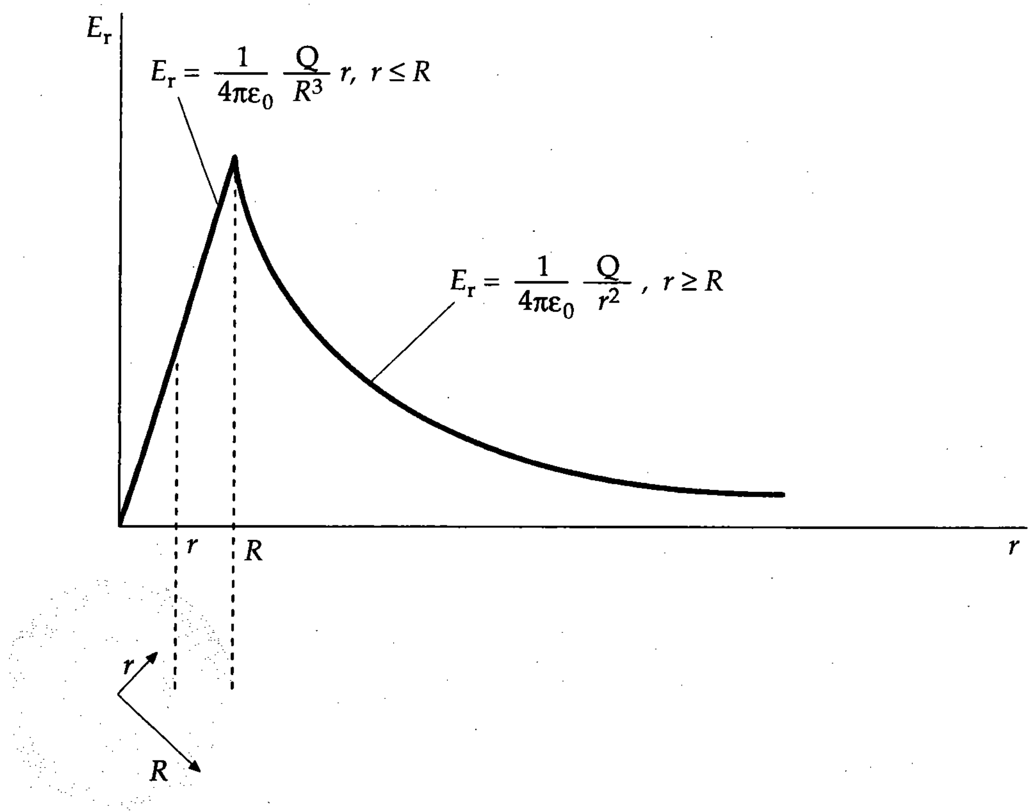




P114 Telegram:

$\bar{E}$ can be calculated directly from ρ (or λ, or Q.)

$$\vec{E} = \int \frac{k\rho}{r^2}\,\hat{r}$$

often This is hard

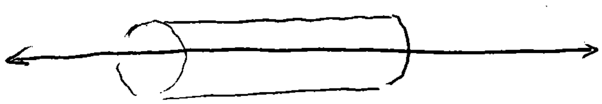
Use Gauss's Law when symmetry allows $|\bar{E}|$ to be Moved out of integral

$$\int \bar{E}\cdot d\bar{a} = Q/\epsilon_0 \longrightarrow |\bar{E}| \int dA = Q/\epsilon_0$$

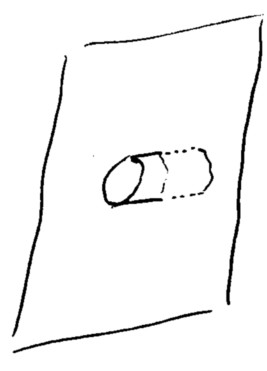
Symmetries often seen

Spherical

Spherical surface



line → cylindrical surface



Plane - pillbox

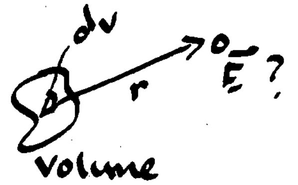
Cylindrical Surface

$\vec{E}$ calculated directly from charge distribution

$$\vec{E} = \int \frac{k\, dQ}{r^2}\, \hat{r}$$

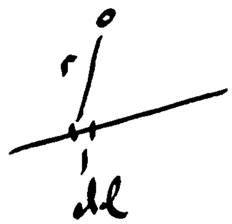
volume

$$\vec{E} = \int_{\text{volume}} \frac{k \rho\, dv}{r^2}\, \hat{r}$$



line

$$\vec{E} = \int_{\text{line}} \frac{k \lambda\, dl}{r^2}\, \hat{r}$$



Surface . . . etc

This is hard !!

Gauss's law allows one to calculate $\vec{E}$ in a much easier way if symmetry allows and you choose the right Gaussian Surface

$$\int_{\text{Gaussian Surf}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

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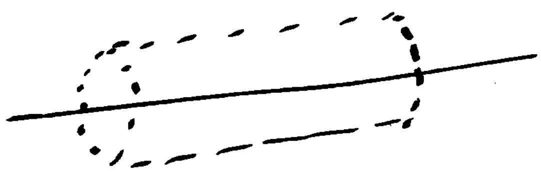
Look at symmetry of charge distribution and choose Gaussian surface so that

$\vec{E} \cdot d\vec{A}$ is easy to evaluate

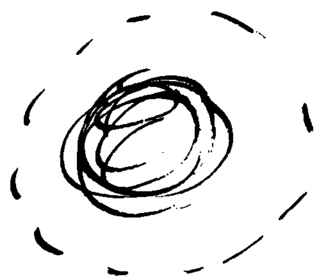
Examples

Point

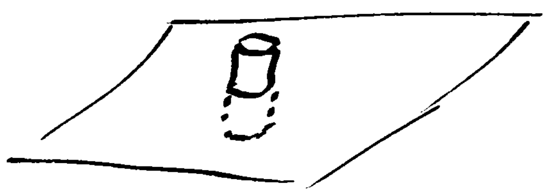
Line



Sphere



Plane



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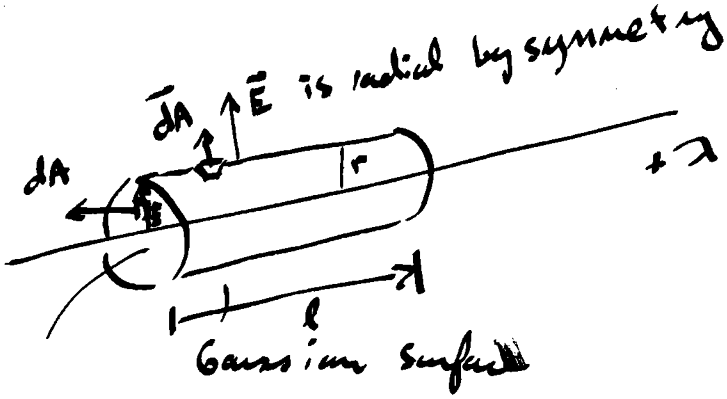
what is $\vec{E}$ for ∞ line charge



$$\int \vec{E}\cdot d\vec{A} = \int_{\text{endcap 1}} \vec{E}\cdot d\vec{A} + \int_{\text{encap 2}} \vec{E}\cdot d\vec{A} + \int_{\text{cylinder}} \vec{E}\cdot d\vec{A}$$

$\vec{E}\cdot d\vec{A} = 0$

$\vec{E} \perp$ to $d\vec{A}$

E is constant at a fixed radius

$$\int \vec{E}\cdot d\vec{A} = |E| \int_{\text{cylinder}} dA = E\, 2\pi r l = \frac{Q_{encl}}{\epsilon_0}$$

by Gauss's law

$Q_{enc} = \lambda l$

$\therefore$

$$E = \frac{\lambda l}{2\pi r l \epsilon_0} = \frac{\lambda}{2\pi r \epsilon_0} \quad \text{radially outward}$$