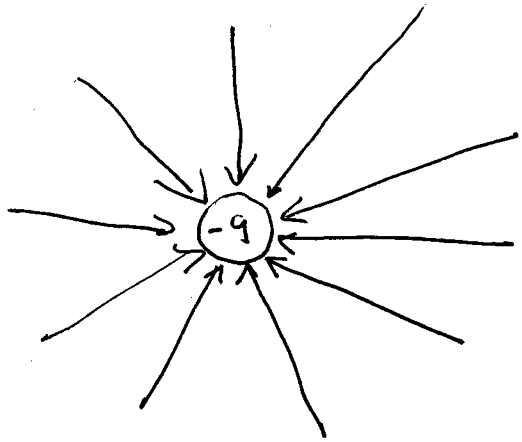
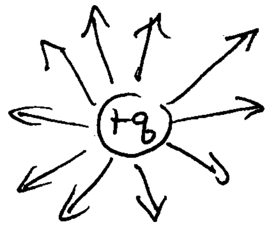
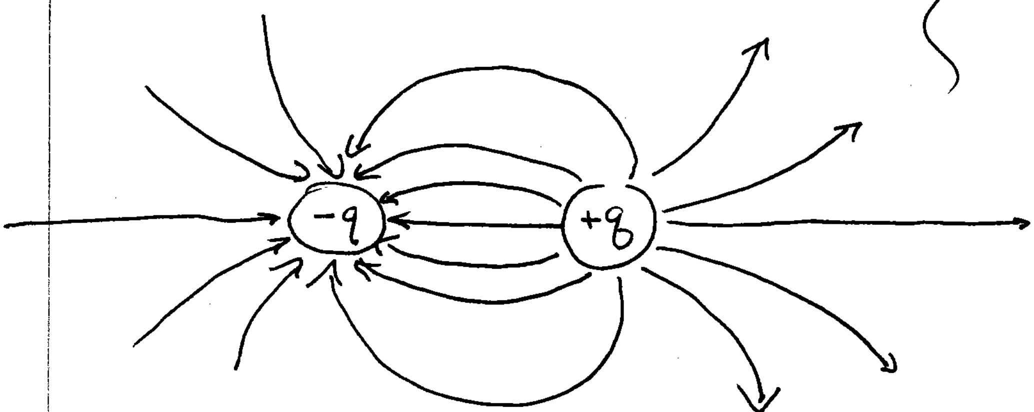
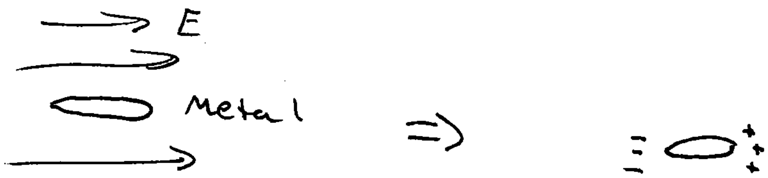


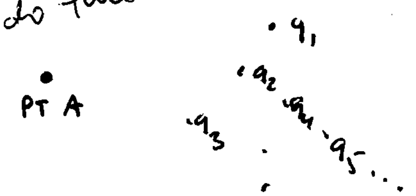

little dipole

orients in field due to torque

⇒ TRANSparency for lines of force

Calculation of The electric Field From Coulombs Law

→ How do I do this




$$\vec{E}_A = \sum_i \frac{\vec{F}_{iA}}{q_0} = \sum \frac{k \frac{q_i q_0 \hat{r}_{iA}}{r_{iA}^2}}{q_0}$$

$$= \sum \frac{k q_i \hat{r}_{iA}}{r_{iA}^2}$$

# Electric Field

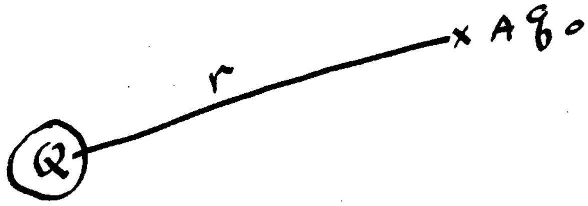
$\vec{E} = \vec{F}/q$ at a point A Vector

$\vec{F}_{\text{on } q \text{ at pt A}} = q\vec{E}$

If you know $\vec{E}$ thruout space you know $\vec{F}$ on charged particles

Electrostatics boils down to determining $\vec{E}$

## $\vec{E}$ surrounding point charge




Place $q_0$ at pt A a distance r from Q

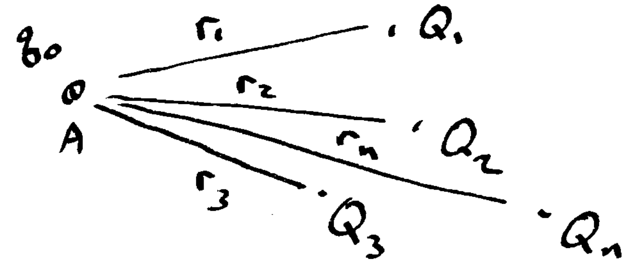
$$\vec{F} = \frac{k q_0 Q}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kQ}{r^2}\hat{r}$$

radially away from Q if Q is +

radially toward Q if Q is −

Electric field strength falls off as $\frac{1}{r^2}$




What is $\vec{E}_A$ for a distribution of discrete point charges at point A

$$\vec{F}_{\text{on test charge}} = \frac{kQ_1 q_0 \hat{r}_1}{r_1^2} + \frac{kQ_2 q_0 \hat{r}_2}{r_2^2} \ldots + \frac{kQ_n q_0 \hat{r}_n}{r_n^2}$$

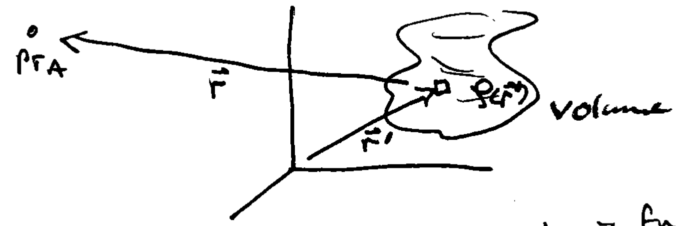
vector sum

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kQ_1}{r_1^2}\hat{r}_1 + \frac{kQ_2}{r_2^2}\hat{r}_2 + \cdots + \frac{kQ_n}{r_n^2}\hat{r}_n$$

overall $\vec{E}$ is vector sum ~~of~~ of all the individual E fields

$\Rightarrow$ Principle of Superposition!

go to Continuous charge distribution




dq is fn of $\vec{r}\,'$

$$d\vec{E} = \frac{k\,dq}{r^2}\,\hat{r} = \frac{k\,\overline{dq(\vec{r}\,')}}{r^2}\,\hat{r}$$

⇒ what is dE due to little dq

$dq = \rho\, dV$ volume charge

$dq = \sigma\, dA$ surface charge

$dq = \lambda\, dL$ line charge

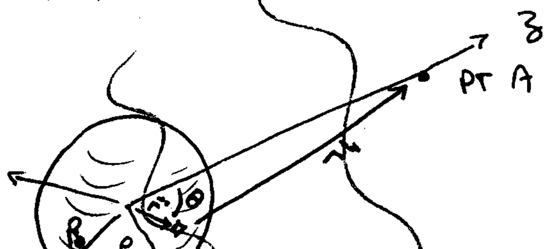
$$E = \int \frac{k\,dq}{r^2}\,\hat{r} \qquad E = \int_{vol} \frac{k\,dq(\vec{r}\,')}{r^2}\,\hat{r}$$

$$= \int_{vol} \frac{k\,\rho(r')\,\hat{r}\,dV}{r^2}$$

NOTE r can change as r' changes in integral

Example

skip for now



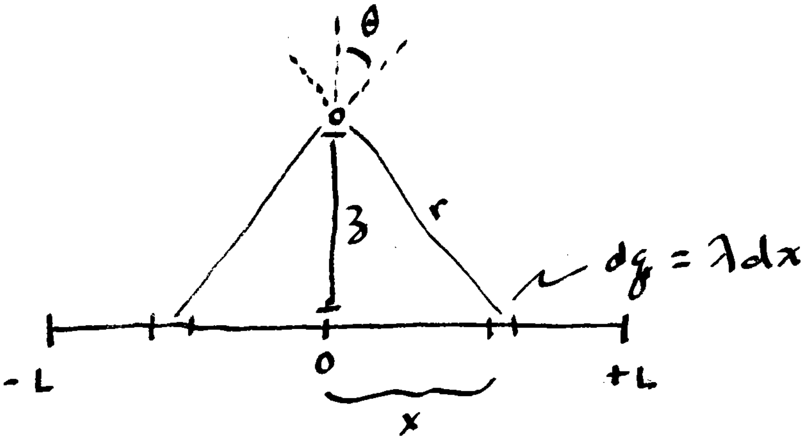

uniform spherical distribution

$$\rho = \frac{Q_{TOT}}{\frac{4}{3}\pi R^3}$$

Calculate $\vec{E}$ along $\hat{z}$ axis outside of a uniform spherical charge dist.

Example

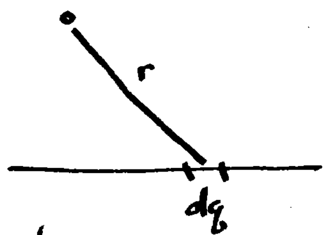
Find $E$ at a distance $z$ above the midpoint of a straight line segment of length $2L$ which carries a uniform line charge of $+\lambda$.




What is the symmetry here?

What direction is $\vec{E}$?

$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$




r varies w/ location of $dq$

$$\vec{dE} = \int_{-L}^{+L} \frac{k\,dq(x)}{r^2}\hat{r}$$

$E_x = |E|\sin\theta$

but notice that x component of E $-L \to 0$ will cancel w/ x component of E from $0 \to L$

- And -

(I) $z$ component of E $-L \to 0$ = $z$ comp of E $0 \to L$

$$E_z = |E|\cos\theta$$

$$d\vec{E} = \cancel{2}\, dE_z\,\hat{z} = dE\cos\theta\,\hat{z}$$

$$\vec{E} = \int_{-L}^{L} \frac{k\, dq(x)}{r^2} \cos\theta\, \hat{z} = \int_{-L}^{L} \frac{k \lambda dx}{r^2} \cos\theta\, \hat{z}$$

$$= (2) \int_{0}^{L} \frac{k \lambda dx}{r^2} \cos\theta\, \hat{z}$$

limits change + mult by 2 due to I

$$r^2 = x^2 + z^2 \qquad \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2+z^2}}$$

$$\vec{E} = (2) \int_0^L \frac{k\lambda}{(x^2+z^2)} \frac{z}{\sqrt{x^2+r^2}}\, \hat{z}$$

$z$ fixed by problem, $x$ is variable we integrate over

$$\vec{E} = 2k\lambda z \int_0^L \frac{dx}{(x^2+z^2)^{3/2}}\, \hat{z}$$

Look up + Evaluate integral

$$= 2k\lambda z \left[ \frac{x}{z^2 (x^2+z^2)^{1/2}} \right]_0^L \hat{z}$$

$$\vec{E} = 2k\lambda \cancel{z} \frac{L}{z^{\cancel{2}}(L^2+z^2)^{1/2}}\, \hat{z}$$

limits

$z >> L$ should look like point charge

$$\vec{E} \rightarrow \frac{2k\lambda L}{z^2} \sim \frac{kQ}{z^2} \,!$$

$L >> \infty$

$$\frac{dL}{dL} = 1$$

$$\frac{d\, zL}{dL} = z$$

$$\vec{E} \rightarrow \frac{2k\lambda}{z}$$

field ~~around~~ around $\infty$ line charge

Is This hard? Yes

Calculating $\vec{E}$ from a charge distribution directly

Much of the next Month will be spent looking at easier ways to get $\vec{E}$

Next topic $\Rightarrow$ Electric flux

Prelude to