Phys 113

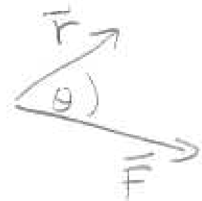
11-14-06

Rotational Motion - review

$\Sigma \vec{\tau} = I\vec{\alpha}$ (like $\Sigma \vec{F} = m\vec{a}$)

$\vec{\tau} = \vec{r} \times \vec{F}$ direction given by right hand rule (RHR)

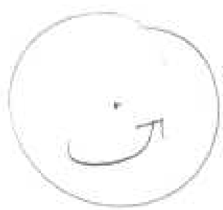
$|\vec{\tau}| = rF\sin\theta$ $(= r_\perp F = rF_\perp)$



θ is angle in between $\vec{r}$ & $\vec{F}$ when you put them tail-to-tail.

$\Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$ $\vec{L} = I\vec{\omega}$ or $\vec{L} = \vec{r} \times \vec{p}$

Right Hand Rules - Practice



what direction is $\vec{\omega}$?

fingers curl in direction of rotation (counter-clockwise), thumb points out.

$\vec{\omega}$ is <u>out</u>

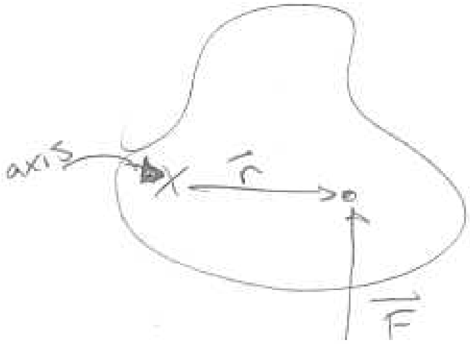
Suppose $|\vec{\omega}|$ is increasing. $|\vec{\alpha}| = \left|\frac{d\vec{\omega}}{dt}\right|$

$\vec{\alpha}$ direction? $\vec{\omega}$ is out and increasing so, $\vec{\alpha}$ <u>out</u>

Suppose $|\vec{\omega}|$ is decreasing. $\vec{\alpha}$?

$\vec{\omega}$ is out and decreasing so $\vec{\alpha}$ <u>in</u>

<u>Do not memorize!</u> Understand instead.

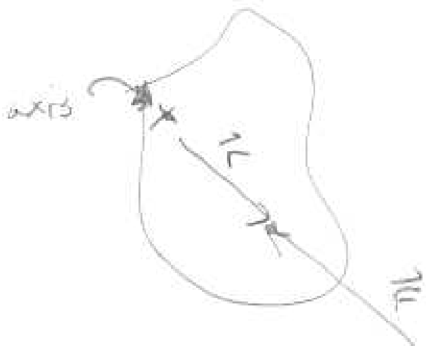


$\vec{F}$ acts in direction shown.

$\vec{r}$ points from axis to where force is acting.

What direction is $\vec{\tau}$? $(\vec{\tau} = \vec{r} \times \vec{F})$

$\vec{r} \times \vec{F} \Rightarrow$ right cross up $\Rightarrow$ $\vec{\tau}$ is out



$\vec{r} \times \vec{F} = 0$ no torque

$(|\vec{r} \times \vec{F}| = rF\sin 180 = 0)$

Get good at RHRs!

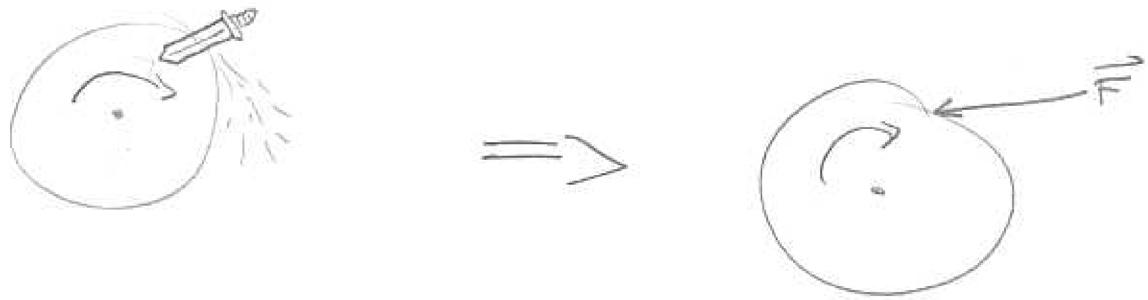
→ use to find directions of $\vec{\omega}$, $\vec{\alpha}$

→ use to find general cross product directions $(\vec{C} = \vec{A} \times \vec{B})$

→ use to find $\vec{\tau} = \vec{r} \times \vec{F}$ directions

(important in Phys 114 too)

Grinding Wheel and Sword



What direction is $\vec{\alpha}$?

up? down? left? right? in? out? none?

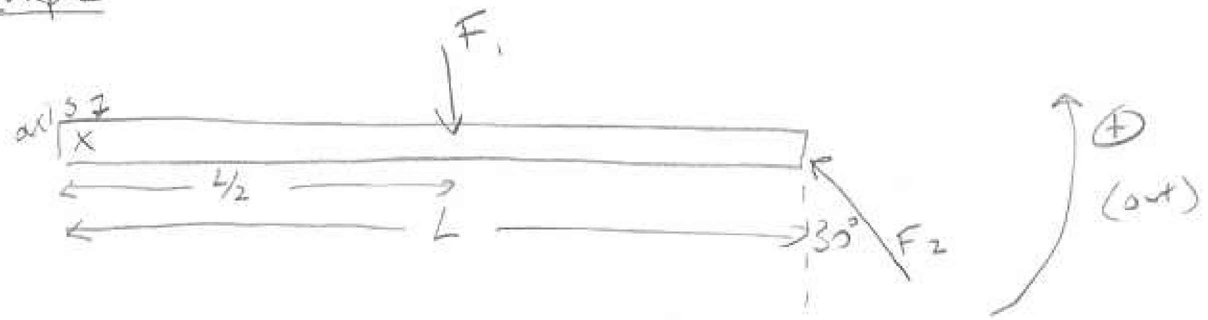
$\boxed{\Sigma \vec{\tau} = I\vec{\alpha}}$ → only $\vec{\tau}$ is caused by $\vec{F}$, so $\vec{\alpha}$ will be in same direction as that $\vec{\tau}$.

$\vec{r} \times \vec{F}$ points out, $\vec{\tau}$ out

$\Sigma \vec{\tau}$ out, so $\vec{\alpha}$ out

\- or - $\vec{F}$ opposes rotation, so it will act to slow it down. $\vec{\omega}$ is pointing in, but now it's decreasing so $\vec{\alpha}$ out

Example



Rod of length L, mass M, is free to rotate about an axis at one end. The forces shown act at a given instant.

$M = 3\,kg$

$L = 2\,m$

$F_1 = 2N$

$F_2 = 6N$

Find $\vec{\alpha}$.

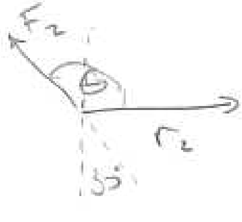
$\Sigma\vec{\tau} = I\vec{\alpha}$ $\qquad I = \frac{1}{3}ML^2$ (book)

$\Sigma\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$

$\vec{\tau}_1$: $\vec{r}_1$, $\vec{F}_1$ $\quad$ in (negative) $\quad |\vec{\tau}_1| = r_1 F_1 = \frac{L}{2} F_1$

(angle b/w is 90°, sin 90 = 1)

$\vec{\tau}_2$: r_2, F_2 $\quad$ out (positive) $\quad |\vec{\tau}_2| = r_2 F_2 \sin\theta_2$



$\theta = 120°$ $\qquad = L F_2 \sin(120°)$

sol $\quad \Sigma\vec{\tau} = -\frac{L}{2}F_1 + LF_2 \sin 120 = I\alpha$

$$\alpha = \frac{-\frac{L}{2}F_1 + LF_2\sin 120}{\frac{1}{3}ML^2}$$

$$\alpha = \frac{-\frac{(2m)}{2}(2N) + (2m)(6N)\sin(120)}{\frac{1}{3}(3kg)(2m)^2}$$

$$= 2.1 \frac{Nm}{kgm^2} \qquad \frac{N\cancel{m}}{kgm^{\cancel{2}}} = \frac{kgm/s^2}{kgm} = 1/s^2 \checkmark$$

$$\boxed{\alpha = 2.1 \,\text{rad}/s^2}$$ positive, so <u>out</u>

<u>Angular Momentum!</u>

what's momentum? $\vec{p} = m\vec{v}$

$$\frac{d\vec{p}}{dt} = \vec{F} \quad \leftarrow \text{Newton's third law}$$

$$d\vec{p} = \vec{F}\, dt$$

to change momentum ($d\vec{p}$) you need to apply a force ($\vec{F}$) for a time (dt)

Linear world | Rotational world

$$x,\ \vec{v},\ \vec{a} \longleftarrow \begin{matrix} s = r\theta \\ v = r\omega \\ a = r\alpha \end{matrix} \longrightarrow \theta,\ \vec{\omega},\ \vec{\alpha}$$

$$\vec{F} \longleftarrow \vec{\tau} = \vec{r} \times \vec{F} \longrightarrow \vec{\tau}$$

$$m \longleftarrow I = \int dm\, r^2 \longrightarrow I$$

$$\Sigma \vec{F} = m\vec{a} \qquad \Sigma \vec{\tau} = I\vec{\alpha}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I\vec{\omega}$$

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} \qquad \Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$$

L is angular momentum. I is "rotational mass"

ω is angular velocity

$\boxed{\vec{L} = I\vec{\omega}}$ for a rotating "rigid body" (like a wheel, not like a tornado)

what about a particle?

$L = I\omega$, $I = mr^2$ for a particle of mass m a distance r from axis

$$\omega = \frac{v}{r}$$

$L = (mr^2)\left(\frac{v}{r}\right) = mrv = r(mv) = rp$

$\boxed{\vec{L} = \vec{r} \times \vec{p}}$ for a particle (like a bullet)

direction given by RHR.

Remember how linear momentum is conserved sometimes?

If $\Sigma F = 0$, $\frac{d\vec{p}}{dt} = 0$, so $\vec{p}_i = \vec{p}_f$

Same with angular momentum.

If $\Sigma \vec{\tau} = 0$, $\frac{d\vec{L}}{dt} = 0$, so $\vec{L}_i = \vec{L}_f$

so $\rightarrow$ when (only when) $\Sigma \vec{\tau}_{ext} = 0$ for a system,

$\boxed{\vec{L}_{i\ system} = \vec{L}_{f\ system}}$

Demo:

$\omega_f > \omega_i$!

When weights in, small I_i

When weights out, large I_f.

but $I_i \omega_i = I_f \omega_f$ (b/c $\vec{L}_i = \vec{L}_f$) so when $I \uparrow$, $\omega \downarrow$

$I_f > I_i$ so $\omega_f < \omega_i$

wheel $\vec{\omega}$ is up.
so system $\vec{\omega}$ is up.
so system $\vec{L}$ is up!

I turn wheel over...

wheel $\vec{\omega}$ is down
but system $\vec{L}$ still needs to be up to conserve!
so everything <u>has</u> to start rotating to get $\vec{L}$ to remain pointing up.

You really have to see the demo to appreciate it...

Next time... <u>Precession!</u>