16 November, 2006

I. Review from Tuesday:

Angular Momentum

$\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} = I\vec{\omega}$

Torque

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = I\vec{\alpha}$

Moment of Inertia

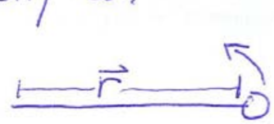
$I = \int r^2 \, dm$

Tie it all together:

$\vec{\tau} = \dfrac{d\vec{L}}{dt} \Rightarrow \sum \vec{L}_{init} = \sum \vec{L}_{final}$ <u>if no external torque applied</u>

II. Conservation of Momentum Examples:

A. Weight orbiting on a string with some orbital velocity $\vec{\omega}$.



$\vec{\omega}$ points <u>out</u> of the page.

Decrease $\vec{r}_i$ and what happens to $\vec{\omega}$?

$$\vec{L}_i = \vec{L}_f$$

$$I_i \vec{\omega}_i = I_f \vec{\omega}_f$$

Recall $I = \int r^2 dm$. If mass stays same and r decreases, then I decreases.

$$\therefore I_i > I_f$$

So $\vec{\omega}_f = \frac{I_i}{I_f} \vec{\omega}_i$. $\frac{I_i}{I_f} > 1 \Rightarrow \vec{\omega}_f > \vec{\omega}_i$.

Weight orbits more quickly.

B. Rotating Student:

Student stands on rotating platform. Rotates with angular speed $\omega_i = 5$ rev/s.

Holds two dumbbells in outstretched arms.
Mass of dumbbells = 5 Kg. = m_d
Length of arm = r_i = .6 m

What happens if student brings dumbbells closer to his chest? Assume final position of dumbbells is $r_f = .2m$. Assume student's moment of inertia, I_s, is $.4\ kgm^2$.

Step 1: Calculate net moment of inertia before and after moving dumbbells.

Before

$$I_i = \Sigma I$$

$$= I_s + I_{m1i} + I_{m2i}$$

Moment of inertia for each dumbbell

$$= .4kgm^2 + 5kg(.6m)^2 + 5kg(.6kg)^2$$

$$= .4kgm^2 + 2 \cdot 1.8\ kgm^2$$

$$= 4\ kgm^2$$

After

$$I_f = I_s + I_{m1f} + I_{m2f}$$

$$= .4kgm^2 + 5kg(.2m)^2 \cdot 2$$

$$= .8\ kgm^2$$

Step 2: Use conservation of angular momentum to calculate ω_f.

$$\vec{L}_i = \vec{L}_f$$

$$I_i \omega_i = I_f \omega_f$$

$$4\,kgm^2 \cdot .5\,\frac{rev}{s} \cdot \frac{2\pi\,rad}{rev} = .8\,kgm^2 \cdot \omega_f$$

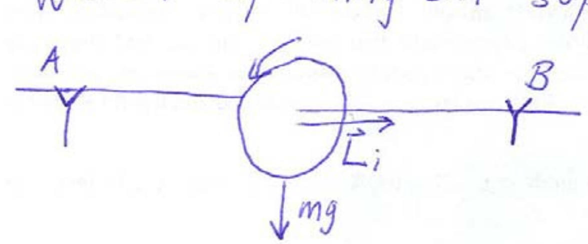
$$\omega_f = \frac{4}{.8} \cdot 3.14\,\frac{rad}{s}$$

$$= 15.7\,\frac{rad}{s} = 2.5\,rev/s$$

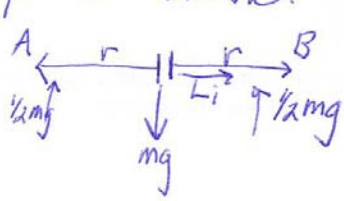
III. Precession

A. Demo Introduction

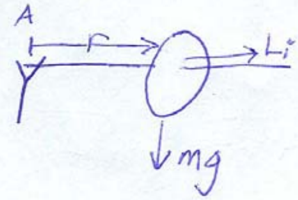
Wheel spinning on supported axis:



Setup is stable.

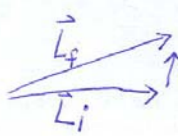


If we remove support on one side, gravity induces a torque around point A:



$\vec{\tau}_1 = \vec{r} \times \vec{F} = \vec{r} \times m\vec{g} = rmg$ into the page
(try right-hand rule!)

Vector diagram <u>from above</u>

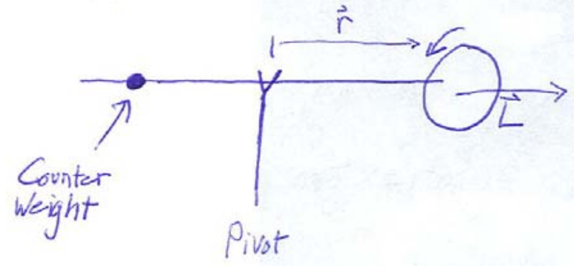


$\uparrow \vec{\tau}_1$ <u>into the page</u>

Notice we have an external torque caused by gravity, so $L_i \neq L_f$. However, we do know $\frac{d\vec{L}}{dt} = \vec{\tau}_1$, so the overall angular momentum vector will now have a component into the page. Wheel will precess counter-clockwise into the page. Slow counter-clockwise rotation is precession.

B. Second Demo - Counter-balanced gyroscope

Heavy wheel rotates on an axis. Axis supported by pivot; opposite end holds counter-weight.



Can adjust position of counter-weight. What happens if:

1. Weight balances gyroscope?: No gravitational torque, no precession.

2. Weight moved close to pivot: Similar to rotating wheel (in previous demo). Gravity pulls downward on gyroscope. Resulting torque induces precession around pivot (Counter-clockwise into the page.).

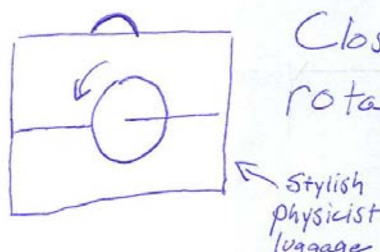
$\xrightarrow{r}$ $\downarrow m\vec{g}$ $\vec{\tau}$ into the page

3. Weight moved far away from the pivot: Force of gravity on weight now > force of gravity on gyroscope. Force on gyroscope now is upward

$\xrightarrow{\vec{r}}$ $\uparrow\vec{F}$ $\vec{\tau}$ out of the page

In case 3, precession is clockwise - opposite of case 2.

C. Final Demo - Gyroscope in a Suitcase



Closed box contains gyroscope rotating around some axis.

What happens if box picked up vertically?
→ Should behave normally.

What if you rotate the box?
→ Precession; box should move in apparently odd way.

Can you deduce initial direction of angular momentum based on how box responds to rotation?