Physics 113 - November 7, 2006

Last Time

$$\boxed{\tau = I\alpha}$$

Angular Acceleration

Torque

moment of inertia

This is the rotational analogue of $F = ma$

Vector aspect of eqn very impt.
but ignored 'till Now.

$\tau \equiv$ torque $\sim rF$ ... more complication to come

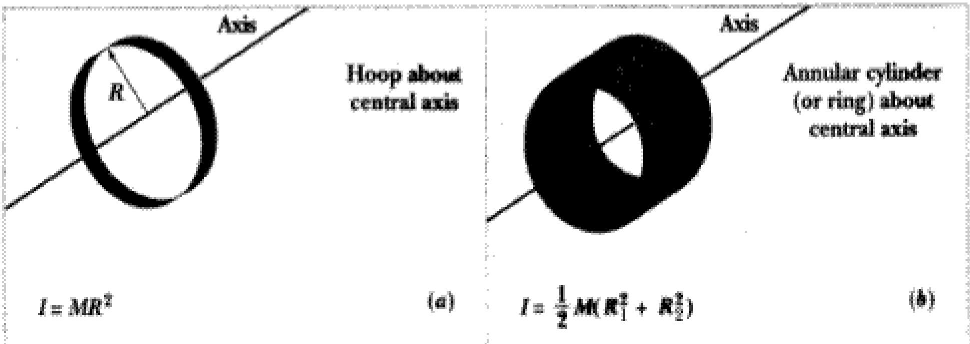
Applied force

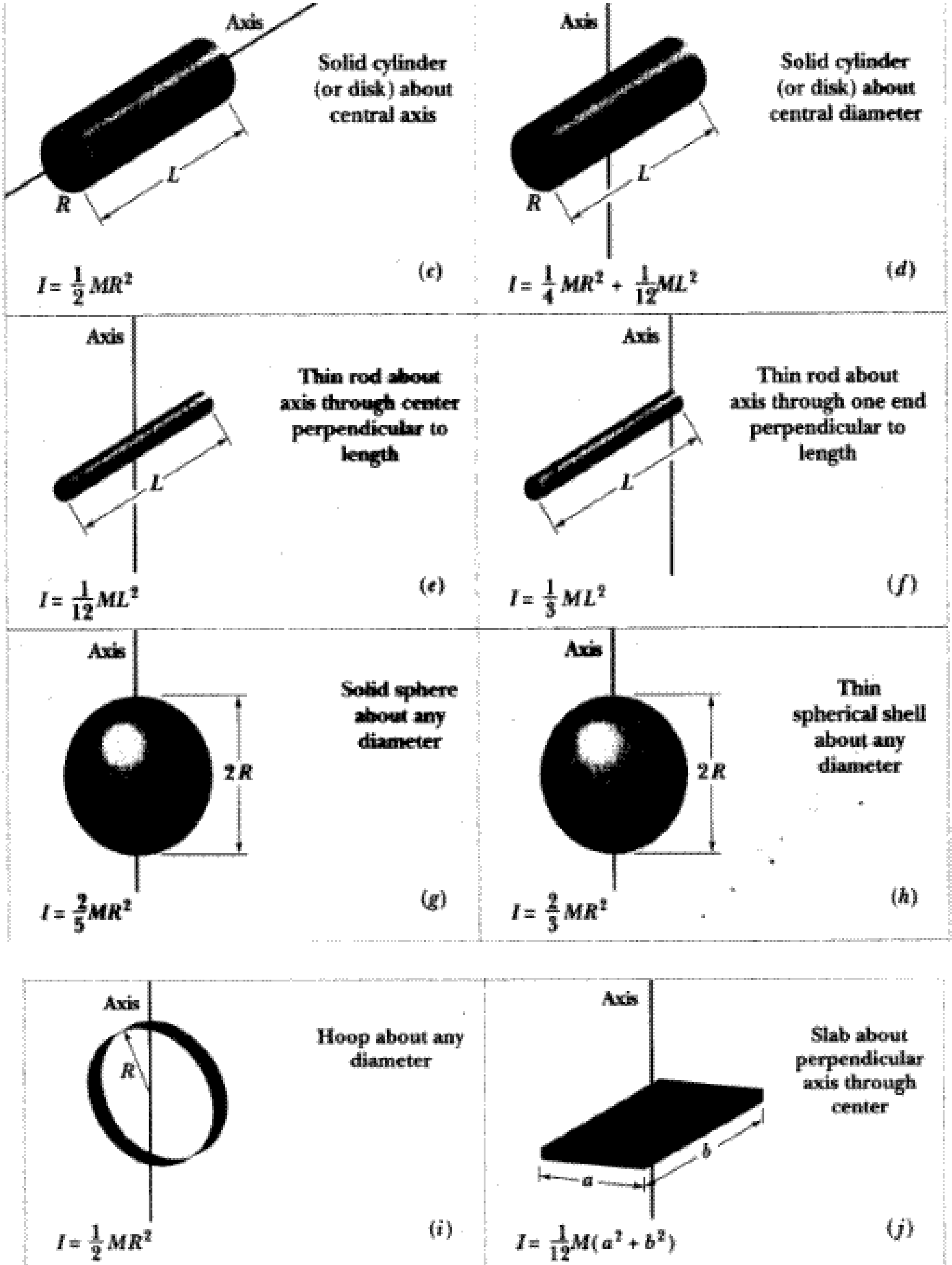
Distance to axis of rotation from pt. of applied force

$\underline{I} \equiv$ moment of Inertia

$$\underline{I} = \int_{\text{volume}} r^2 \, dm = \int_{\text{volume}} r^2 \rho \, dv$$

Many I are already calculated for you
can look up in a table

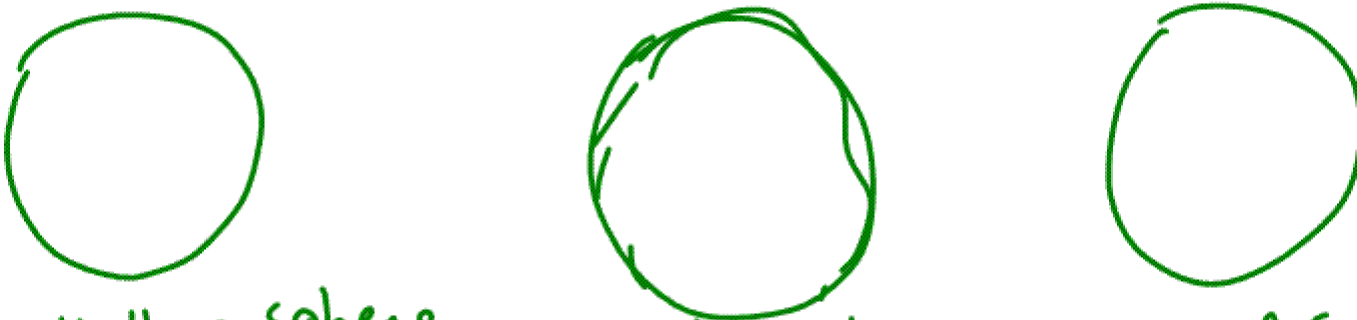




$I = \int r^2 dm$

As more of mass is farther from Axis of Rotation

$I \rightarrow$ larger



Hollow sphere $I = \frac{2}{3} MR^2$

Wheel $I = MR^2$

Solid sphere $I = \frac{2}{5} MR^2$

How far up ramp does cyl. climb?

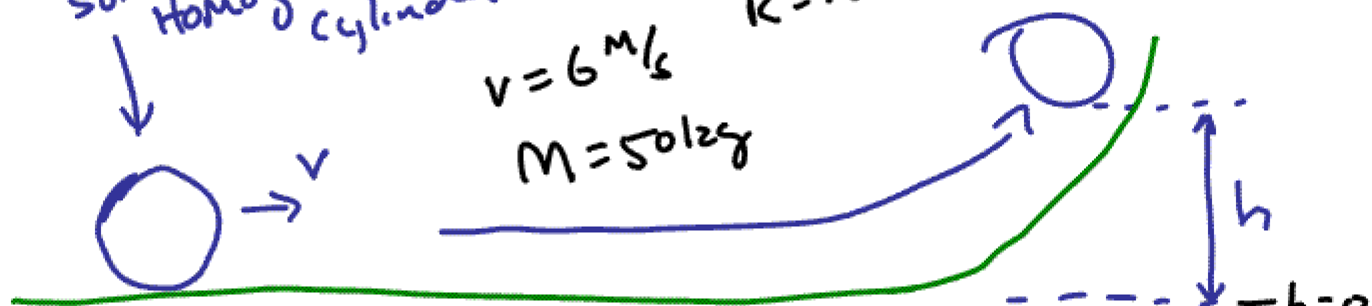
Solid Homogenous cylinder

NOT slipping

$v = 6\ m/s$

$M = 50\ kg$

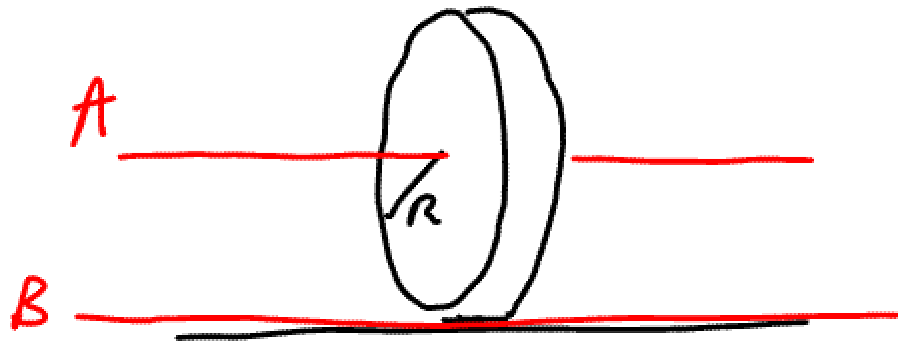
$R = 15\ cm$



E_{start} E_{end}

$$\cancel{PE}_{0} + KE_{translation} + KE_{rotation} = mgh$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = mgh$$



$$I_A = \frac{1}{2}MR^2$$ (from table)

$$I_B = I_A + Mr^2$$ parallel axis theorem

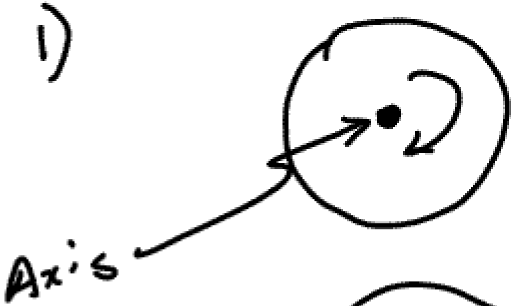
Dist of Axis B from Axis A

Axes //

$$I_B = I_A + MR^2$$

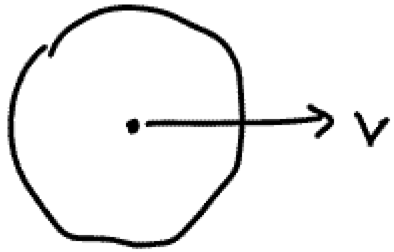
$$I_B = \frac{3}{2}MR^2$$

1)



rotation abt C.M.

$$KE = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

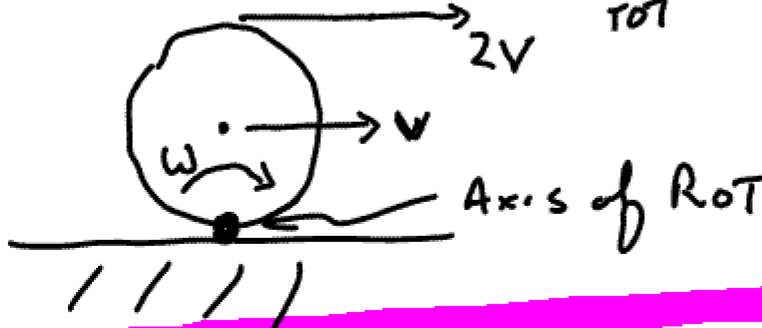


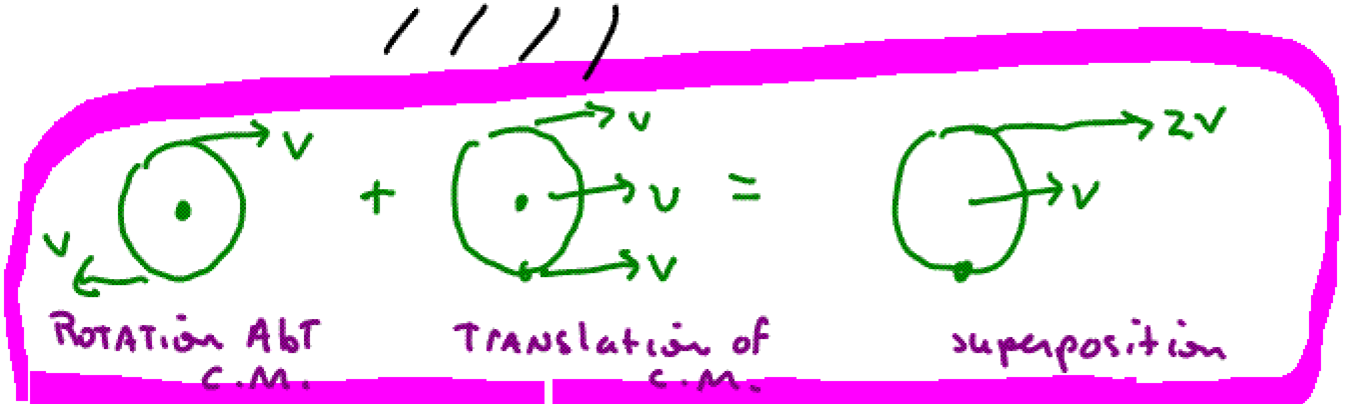
Moving C.M $\quad KE = \frac{1}{2}Mv^2$

$$KE_{TOT} = KE_T + KE_{ROT}$$

$$KE_{TOT} = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$$

2)





2) Body rotating abt Axis at bottom

C.M. NOT moving

$$KE_{TOT} = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{3}{2} MR^2\right)\omega^2$$

$$\frac{v^2}{R^2}$$

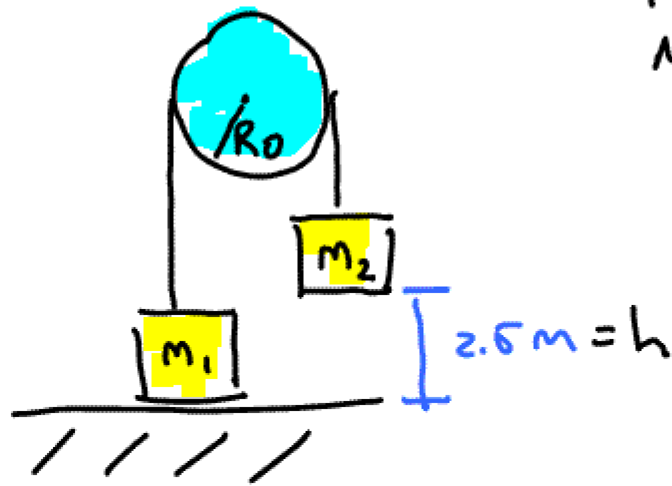
$$KE_{TOT} = \frac{3}{4} Mv^2$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$

$$\frac{3}{4}\cancel{M}v^2 = \cancel{M}gh$$

$$h = \frac{3}{4}\frac{v^2}{g}$$

10-65 Giancoli



$M_1 = 35.0$ kg
$M_2 = 38.0$ kg

Pulley: uniform cylinder w/ radius $R_0 = 0.3$m and Mass $M = 4.8$kg

Init M_1 on ground M_2 at rest 2.5 m above grnd

Assume rope massless and does Not Slip

System released → what is speed of M_2 just before it hits the ground?

Two ways to solve this problem → Energy conservation
→ Newton's Laws

Energy Conservation

$\uparrow v \qquad \downarrow v$

$$E_{START} = E_{end}$$

$$m_2 g h = m_1 g h + \tfrac{1}{2} M_1 v_1^2 + \tfrac{1}{2} M_2 v_2^2 + \tfrac{1}{2} I \omega^2$$

$$v_1 = v_2 = v = R_0 \omega$$

$$I = \tfrac{1}{2} M R_0^2$$

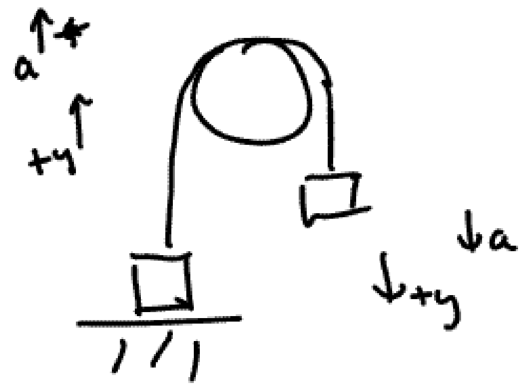
$$M_2 g h = M_1 g h + \tfrac{1}{2}(M_1 + M_2) v^2 + \tfrac{1}{2}\left(\tfrac{1}{2} M R_0^2\right)\frac{v^2}{R_0^2}$$

Solve for v

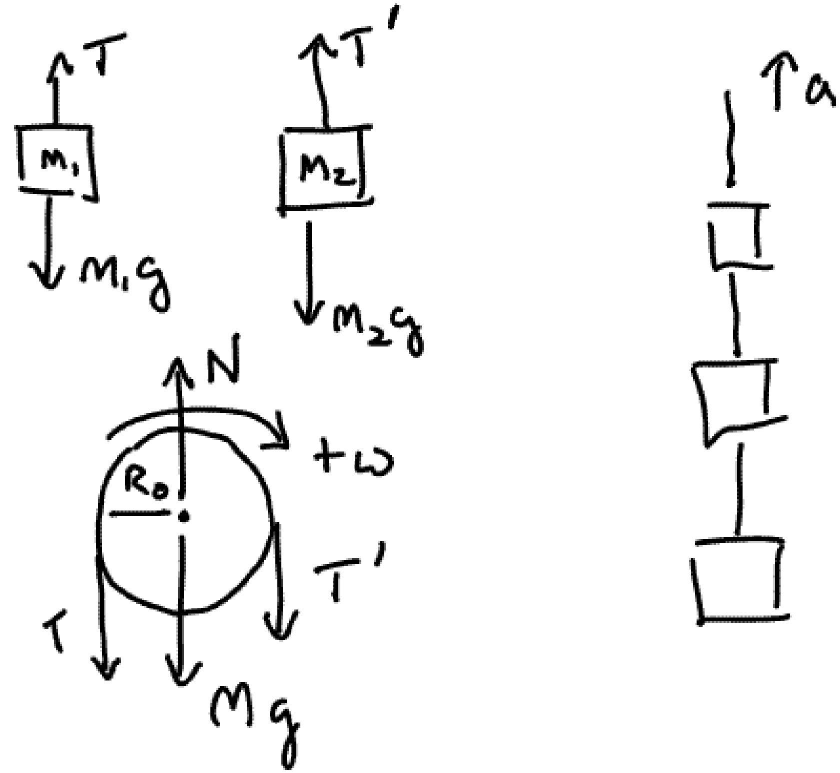
$$v = \pm \sqrt{\frac{(M_1 - M_2) g h}{\frac{M}{4} + \frac{M_2}{2} + \frac{M_1}{2}}} = \pm 1.4 \, m/s$$

$1.4 \, m/s$

using Newton's Laws



FBD's



With massive pulley

$T \neq T'$

M_1

$\sum F_y = M_1 a$

$M_1 a = T - M_1 g$

$\uparrow$

M_2

$\sum F_y = M_2 a$

$M_2 a = M_2 g - T'$

$\uparrow$

Pulley

$\sum \tau = I\alpha$

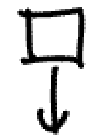
$R_0 T' - R_0 T = I \dfrac{a}{R_0}$

$\frac{1}{2} M R_0^2$

3 eqns 3 unknowns

$$a = \frac{(m_2 - m_1)\,g}{\frac{1}{2}M + m_1 + m_2}$$

const a



RH mass

$$a = 0.4 \ \mathrm{m/s^2}$$

$$v_2^2 = \cancelto{0}{v_{02}^2} + 2ah$$

$$v_2^2 = (2)(.4)(2.5) = 2$$

$$v_2 = 1.4 \ \mathrm{m/s}$$

Both methods give the same answer.