Physics 113 - October 31, 2006

Happy Halloween

Last Time:

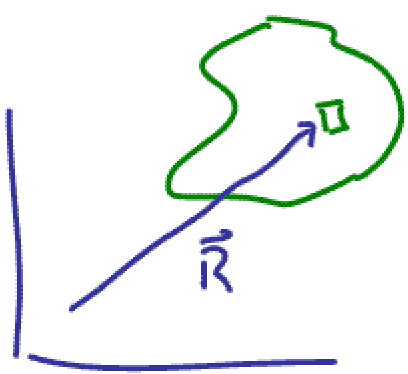
Center-of-Mass coordinates

Mass Weighted Average Position

$$x_{c.m.} = \frac{\sum m_i x_i}{\sum m_i} \quad \text{-or-} \quad \frac{\int x\,dm}{\int dm} \qquad dm = \rho\,dV$$

$$y_{c.m.} = \frac{\sum m_i y_i}{\sum m_i} \quad \text{-or-} \quad \frac{\int y\,dm}{\int dm}$$

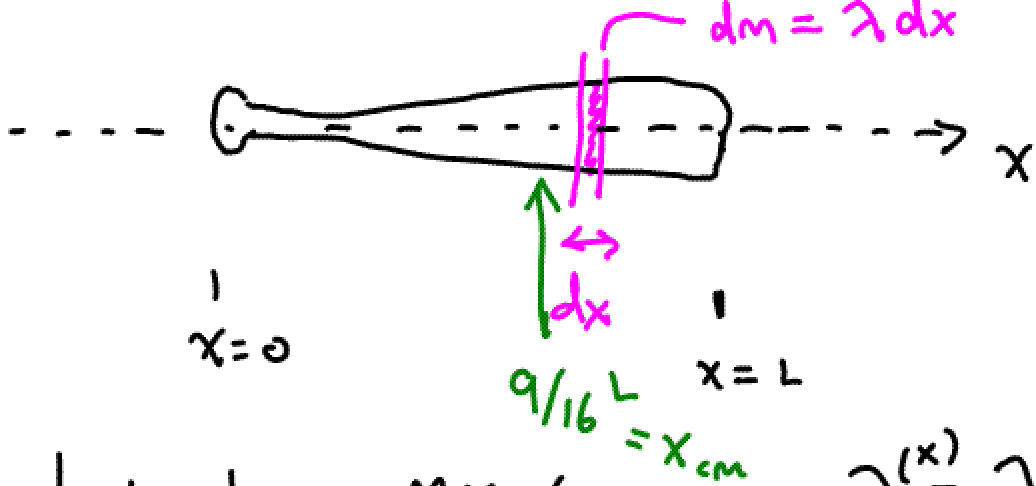
$$z_{c.m.} = \frac{\sum m_i z_i}{\sum m_i} \quad \text{-or-} \quad \frac{\int z\,dm}{\int dm}$$



discrete | continuous

Example

find C.M. along x



$\lambda \equiv$ linear
$\sigma \equiv$ area
$\rho \equiv$ volume
densities

bat has $\text{Mass}/\text{Length} = \lambda(x) \equiv \lambda_0\left(1 + \frac{x^2}{L^2}\right) \quad 0 \le x \le L$

"Linear" Mass Density

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x\,\lambda(x)\,dx}{\int_0^L \lambda\,dx}$$

$$M = \int dm = \int_0^L \lambda\,dx = \int_0^L \lambda_0\left(1 + \frac{x^2}{L^2}\right)dx = \int_0^L \lambda_0\,dx + \int_0^L \frac{\lambda_0 x^2}{L^2}\,dx$$

$$= \lambda_0 x \Big|_0^L + \frac{\lambda_0}{L^2}\frac{x^3}{3}\Big|_0^L = \lambda_0 L + \frac{\lambda_0}{L^2}\frac{L^3}{3}$$

$$= \lambda_0 L + \frac{\lambda_0 L}{3} = \frac{4}{3}\lambda_0 L$$

$$\int x\,dm = \int_0^L \lambda_0\left(1 + \frac{x^2}{L^2}\right) x\,dx = \int_0^L \lambda_0 x\,dx + \int_0^L \lambda_0 \frac{x^3}{L^2}\,dx$$

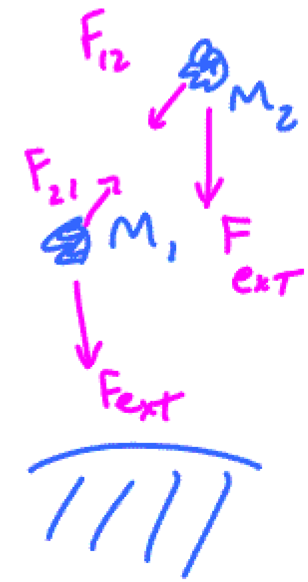
$$= \lambda_0 \frac{x^2}{2}\Big|_0^L + \frac{\lambda_0}{L^2}\frac{x^4}{4}\Big|_0^L = \frac{\lambda_0 L^2}{2} + \frac{\lambda_0}{L^2}\frac{L^4}{4}$$

$$= \frac{\lambda_0 L^2}{2} + \frac{\lambda_0 L^2}{4} = \frac{3}{4}\lambda_0 L^2$$

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\frac{3}{4}\lambda_0 L^2}{\frac{4}{3}\lambda_0 L} = \frac{9}{16} L$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n$$

$$F_{21} = -F_{12}$$



$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt}$$

$$M\vec{V}_{cm} = m_1\vec{v}_1 + \cdots + m_n\vec{v}_n$$

$$M\vec{a}_{cm} = m_1\vec{a}_1 + \cdots + m_n\vec{a}_n$$

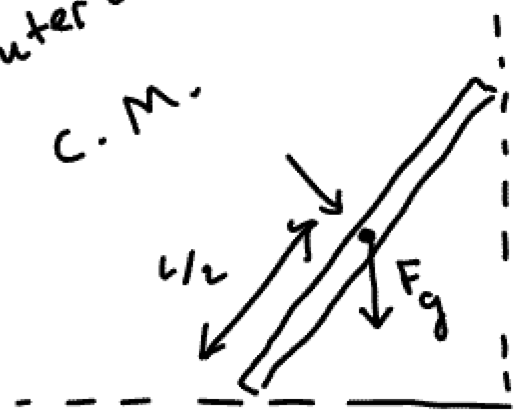
$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 \cdots + \vec{F}_n = \Sigma F_{ext} + \Sigma F_{int}$$

0

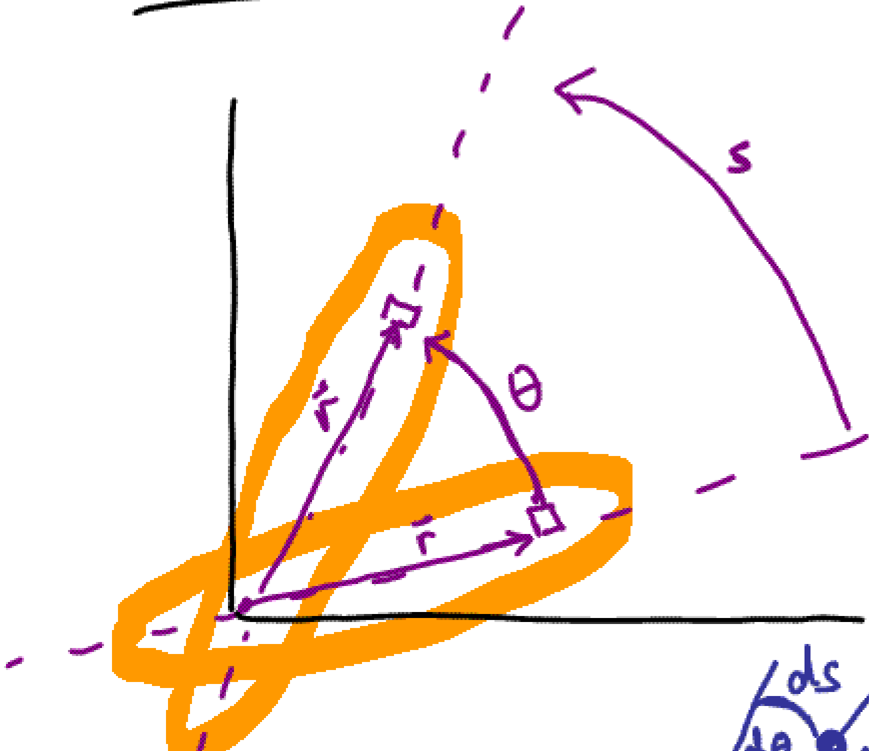
$F_{ext} + F_{21}$ $\qquad$ $F_{ext} + \underbrace{F_{12}}_{-F_{21}}$

Center of gravity

C.M.



Rotational Kinematics

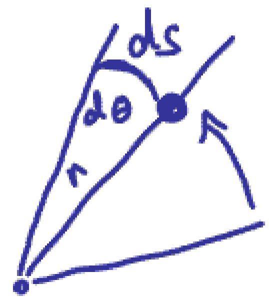


$S = r\theta$

Arclength = r (radians)

$S = r\theta$

$ds = r\,d\theta$



$$\frac{ds}{dt} = r\frac{d\theta}{dt} \quad \omega$$

Tangential Linear velocity

m/s

Angular velocity

$\frac{\text{radians}}{s}$

Tangential linear m/s^2 acceleration → $\frac{d^2s}{dt^2} = r\frac{d^2\theta}{dt} \quad \alpha$

Angular Accel rad/s^2

$$s = r\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow v = r\omega$$

$$\frac{d^2s}{dt^2} = r\frac{d^2\theta}{dt^2} \rightarrow a = r\alpha$$

$$\frac{d\theta}{dt} = \omega$$

$$d\theta = \omega\, dt$$

$$\int d\theta = \int \omega\, dt$$

$$\boxed{\theta - \theta_0 = \int \omega\, dt}$$

General

$$\frac{dx}{dt} = v$$

$$dx = v\, dt$$

$$\int dx = \int v\, dt$$

$$\boxed{x - x_0 = \int v\, dt}$$

$$\frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha\, dt$$

$$\int d\omega = \int \alpha\, dt$$

Analogue to $v - v_0 = \int a\, dt$

$$\boxed{\omega - \omega_0 = \int \alpha\, dt}$$

Assumption

constant α

$$\omega - \omega_0 = \int \alpha\, dt = \alpha \int_{t_0=0}^{t} dt$$

$v = v_0 + at$

$r\omega = r\omega_0 + r\alpha t$

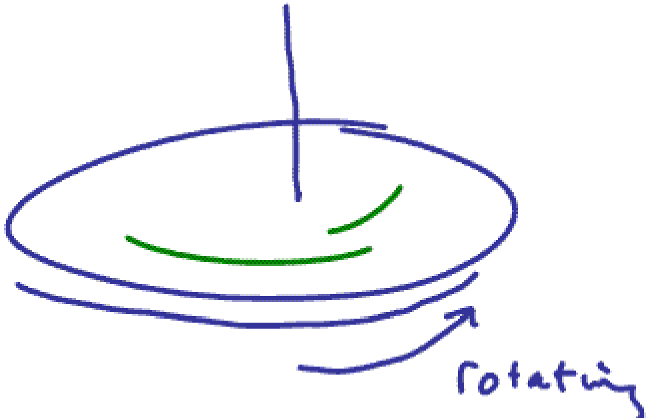
$$\boxed{\omega = \omega_0 + \alpha t}$$

const α

$x = x_0 + v_0 t + \frac{1}{2} a t^2 \longrightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$v^2 = v_0^2 + 2a(x - x_0) \longrightarrow \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$x - x_0 = \frac{1}{2}(v + v_0)t \longrightarrow \theta - \theta_0 = \frac{(\omega + \omega_0)}{2} t$



Disk

Initially rotating at 120 rad/s

Slows down w/ Const. Angular Acceleration of 4.0 rad/s^2

How much time elapses before disk stops rotating?

$\omega = \omega_0 + \alpha t$

$0 = 120 - (4.0)t$

$t = 30$ seconds

$F = ma$

torque

moment of inertia

$\sim mr^2$

$(rF) = (\quad)$ ang. Accel