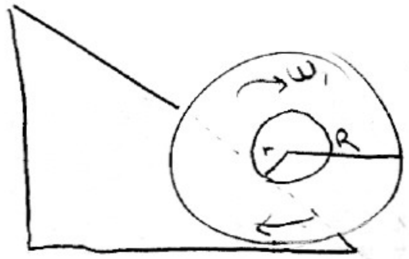
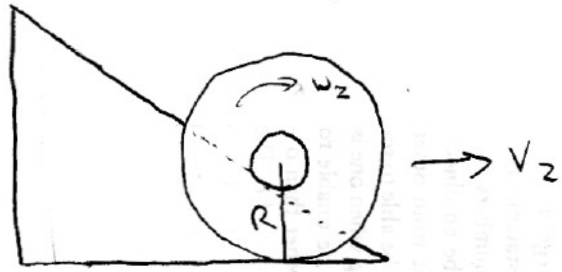


Just before it hits the ground



Just after it hits the ground

Why does the dumbell take off with greatly increased translational speed when the disks come in contact with the ground? (why is $v_2 > v_1$?)

If the ground were frictionless, the dumbell would roll <u>and</u> slip, and v would <u>not</u> increase once it hit the ground. So we cannot ignore friction from the floor. Because this friction causes a torque, and $\vec{\tau} = \frac{d\vec{L}}{dt}$, angular momentum is not conserved.

However! Do not be discouraged! Our only forces are concervative! Your book told you (and others probably told you too) that friction is not a conservative force, but thats not entirely true. Kinetic friction isn't, but static friction is. (it is path-independent b/c there is no path - static friction acts over <u>no</u> distance.) So, because all of our forces are conservative, mechanical energy is conserved.

So if we look at our dumbell just before and just after the disks come in contact with the ground, we can say that mgh is zero in both cases, so KE is conserved. (rotational + translational)

$$KE_i = KE_f$$

$$\frac{1}{2}I\omega_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}mv_2^2$$

"I" does not change, (it's still rotating about the same axis), so b/c $\vec{L} = I\vec{\omega}$ does change (b/c of friction.... net torque) we know that "ω" does change. That's why I labeled them ω_1 & ω_2.

Replacing ω_1 with $\left(\frac{v_1}{r}\right)$

and ω_2 with $\left(\frac{v_2}{R}\right)$....

$$\frac{1}{2}I\frac{v_1^2}{r^2} + \frac{1}{2}mv_1^2 = \frac{1}{2}I\frac{v_2^2}{R^2} + \frac{1}{2}mv_2^2$$

grouping...

$$v_1^2\left(\frac{1}{2}I\frac{1}{r^2} + \frac{1}{2}m\right) = v_2^2\left(\frac{1}{2}I\frac{1}{R^2} + \frac{1}{2}m\right)$$

solving for v_2^2...

$$v_2^2 = v_1^2\left[\frac{\left(\frac{1}{2}I\frac{1}{r^2} + \frac{1}{2}m\right)}{\left(\frac{1}{2}I\frac{1}{R^2} + \frac{1}{2}m\right)}\right]$$

because $r<R$, the numerator is larger than the denominator, $[\quad] > 1$, so $v_2^2 > v_1^2$ which means that $v_2 > v_1$