

We will do more w/ torque when we get to static equilibrium.

Angular momentum

What do we mean by Momentum?

$$\frac{dp}{dt} = F \quad dp = F dt$$

To change momentum you must exert a force for a time.

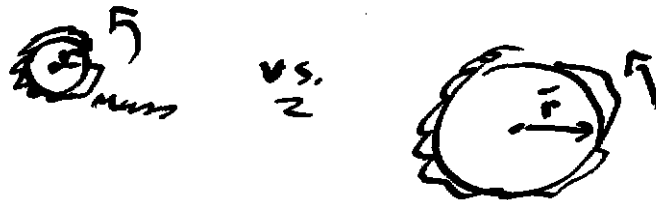
if Momentum is large ... must exert a larger force to stop an object in same time interval

What is Angular ~~momentum~~ analogue of Momentum?

$$p = mv \quad v = r\omega$$

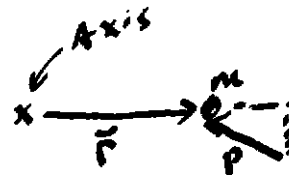
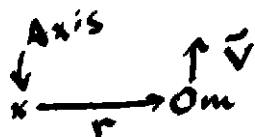
$$\text{Angular Momentum} \sim I\omega$$

$$\sim mr^2\omega \sim r m r\omega \sim r m v \sim r p$$



Angular Momentum is a vector $\equiv \vec{L}$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = I\vec{\omega} \quad \text{same direction as } \vec{\omega}$$



Again \Rightarrow cross product projects out the component of \vec{p} \perp to \vec{r}

$$\vec{L} = \vec{r} \times \vec{p}$$

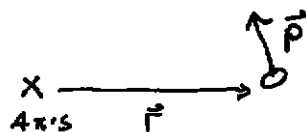
$$\frac{d\vec{L}}{dt} = \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}} \times \vec{p} + \vec{r} \times \underbrace{\frac{d\vec{p}}{dt}}_{\vec{F}}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

Rate of change of Angular momentum =
torque of net force acting on object

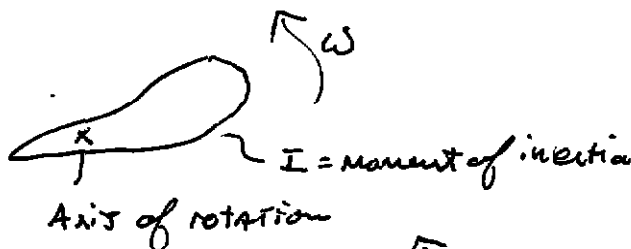
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Angular Momentum



$$\vec{L} = \vec{r} \times \vec{p}$$

discrete body ↗



$$\vec{L} = I \vec{\omega}$$

extended rigid body ↗

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

(just like $\frac{d\vec{p}}{dt} = \vec{F}$ in linear case)

In the absence of $\vec{\tau}_{NET} = 0$ $\frac{d\vec{L}}{dt} = 0$

Also, ~~the~~ Angular Momentum is conserved just as in Linear Momentum.

$$\sum \vec{L}_{NET} = \sum \vec{L}_{Final}$$

Problem 10 (10 pts) – zero/half/full credit:

The matter in stars is in equilibrium between the gravitational force pulling in radially and the "radiation pressure" of energy released by thermonuclear reactions pushing out. When they run out of nuclear fuel in the center (core), the radiation pressure is reduced and they collapse.

Under gravitational collapse, the radius of a spinning spherical star of uniform density shrinks by a factor of 2, with the resulting increased density remaining uniform throughout as the star shrinks. What will be the ratio of the final angular speed ω_2 to the initial angular speed ω_1 ? Select the correct answer below. You must show your supporting work to get credit.

$\omega_2/\omega_1 =$ (a) 2 (b) 0.5 (c) 4 (d) 0.25 (e) 1.0

Angular Momentum is conserved

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$I \text{ for solid sphere} = \frac{2}{5} MR^2$$

$$\frac{2}{5} MR_i^2 \omega_i = \frac{2}{5} MR_f^2 \omega_f$$

$$R_i^2 \omega_i = \left(\frac{1}{2} R_i\right)^2 \omega_f$$

$$\frac{\omega_f}{\omega_i} = 4$$

Just as in linear momentum

Angular Momentum is Conserved!

$$\sum \vec{L}_{\text{initial}} = \sum \vec{L}_{\text{final}}$$

Let's do a couple of example problems to get the hang of some of this New Stuff —

Example

~~An ice skater with out stretched hands spins with angular velocity $\omega = 0.60 \text{ rev/s}$~~



A student stands on a turntable with dumbbells held in outstretched hands.

Initially the student is given an angular velocity of 0.5 revolutions/second.

The student then pulls the weights in close to his/her body. What is the student's angular velocity in the new configuration?

Given info M of each dumbbell = 5 kg

Length of arm (to center of chest) $\sim 0.6 \text{ m}$

$I_{\text{body}} \sim 0.40 \text{ kg}\cdot\text{m}^2$

Masses held close ... dist from body center = ~~0.2 m~~ ^{0.2 m}

Use momentum conservation:

$$L_{\text{initial}} = L_{\text{final}} \rightarrow \sum I\omega_{\text{initial}} = \sum I\omega_{\text{final}}$$

$$(I_{\text{body}} + I_{\text{mass1}} + I_{\text{mass2}}) \omega_{\text{initial}} = (I_{\text{body}} + I_{\text{mass1}} + I_{\text{mass2}}) \omega_{\text{final}}$$
$$[0.4 \text{ kg}\cdot\text{m}^2 + (5 \text{ kg})(0.6 \text{ m})^2 \cdot 2] 0.5 \text{ rev/s} \cdot 2\pi \frac{\text{rad}}{\text{rev}}$$

$$= [0.4 \text{ kg}\cdot\text{m}^2 + (5 \text{ kg})(0.2 \text{ m})^2 \cdot 2] \omega_{\text{final}}$$

$$12.56 \text{ rad/s} \cdot \text{kg}\cdot\text{m}^2 = 0.8 \text{ kg}\cdot\text{m}^2 \omega_{\text{final}}$$

$$\omega_{\text{final}} = 15.7 \text{ rad/s} = 2.4 \text{ rev/s}$$

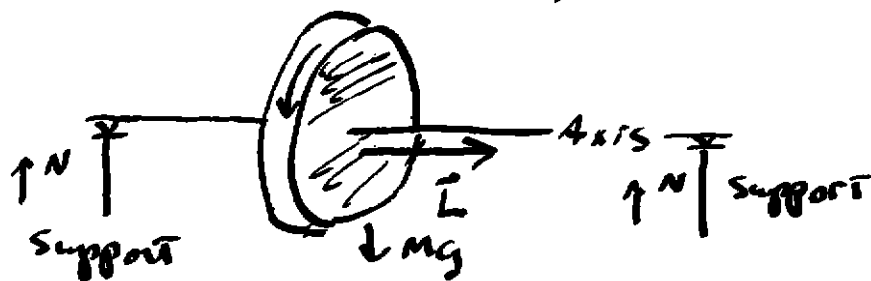
Precession and Gyroscopes

recall

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$



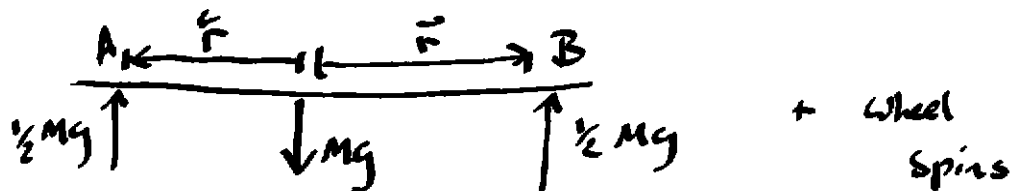
heavy disc spinning on an axis



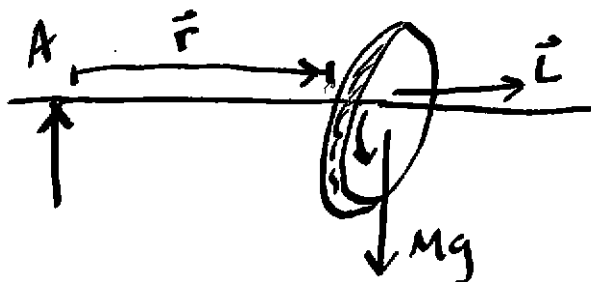
heavy wheel spinning on horizontal axis

~~Take away one support~~

System is STABLE



Now Take away one support (remove support B)



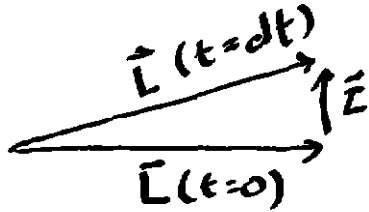
Now there is a Torque about point A

$$\vec{\tau}_A = \vec{r} \times \vec{F} = \vec{r} \times M\vec{g} = rmg \text{ into page}$$

Now recall

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

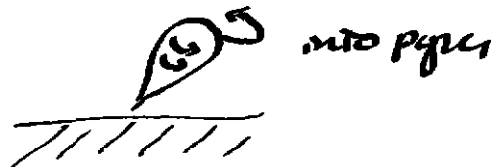
view from above



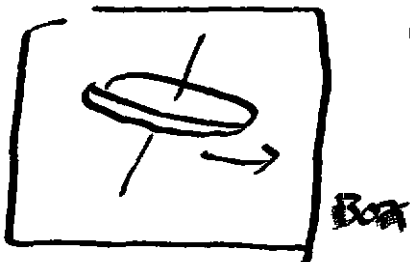
\Rightarrow Axis + spinning disk supported at one end and free to rotate about support does NOT fall \Rightarrow IT rotates in the horizontal plane!!

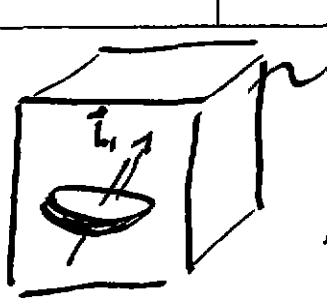
Is this weirdness or what!?

This is known as precession
Have you ever noticed a Top spinning?
It precesses



Imagine a system w/ a disk free to spin ~~in~~ 3d
along an axis in 3d
i.e. Axis is free to rotate



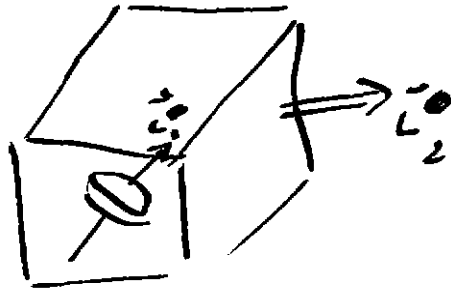


$$\vec{L}_2 = 0$$

suppose box is rotated
what happens?



Total \vec{L} is conserved



$$\text{if } \vec{L}_2 \neq 0$$

\vec{L}_1 must ~~change~~
change
to

cancel out \vec{L}_2 in
vector sum!

There is a change in the orientation of the
spinning disk in response to the rotation of
the box (about Any Axis)

Gyroscopes are a priceless tool for navigation