Historical Application of Momentum conservation

The Neutrino ~1930

β decay

Expect

$$n \rightarrow p + e^-$$ 2 body decays

or

$$p \rightarrow n + e^+$$

Total energy available

$$Q = K_1 + K_2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} M_2 V_2^2$$

P cons. $\quad M_1 V_1 = -M_2 V_2$

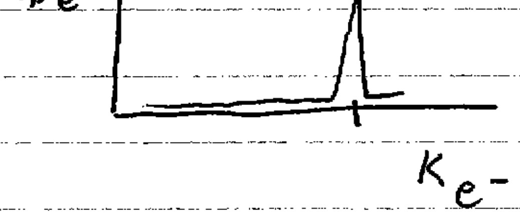
$$\tfrac{1}{2} m_1^2 v_1^2 = \tfrac{1}{2} M_2^2 V_2^2$$

$$M_1 K_1 = M_2 K_2$$

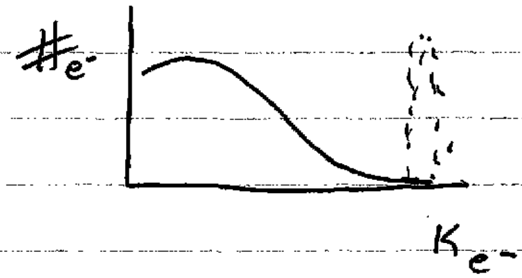
$$K_1 = \frac{M_2}{M_1 + M_2} Q \qquad K_2 = \frac{M_1}{M_1 + M_2} Q$$

For a specific reaction Q is fixed, M_1, M_2 also fixed

expect



Actually see

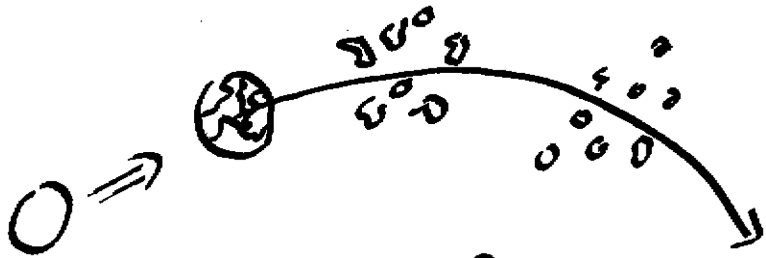


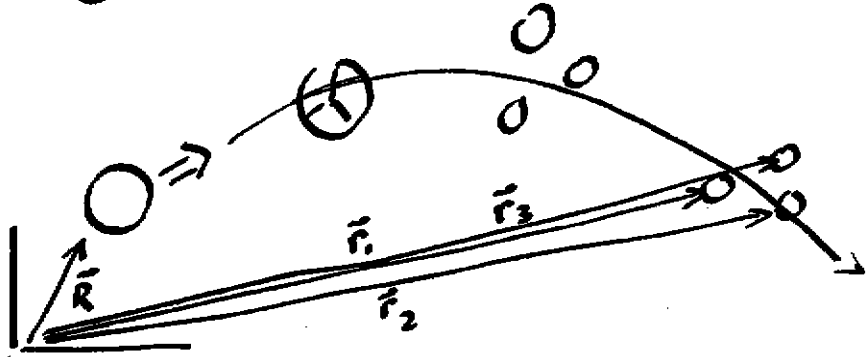
Why? $\Rightarrow$ 3 body decay

$$n \rightarrow p + e^- + \bar{\nu}$$

Pauli suggested the neutrino

Center-of-Mass





An object moving along at position $R(t)$ breaks into 3 (or more) objects w/ positions described by vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$

Momentum Conservation

$$\Longrightarrow \quad \Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt}$$

$$\frac{d}{dt}(M\vec{R}) = \frac{d}{dt}(m_1\vec{r}_1) + \frac{d}{dt}(m_2\vec{r}_2) + \frac{d}{dt}(m_3\vec{r}_3)$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3$$

break into component

$$\begin{cases} MX = m_1x_1 + m_2x_2 + m_3x_3 \\ MY = m_1y_1 + m_2y_2 + m_3y_3 \\ MZ = m_1z_1 + m_2z_2 + m_3z_3 \end{cases}$$

-or-

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$$

Similar for Y, Z

These are Known as Center-of-Mass coordinates

Can consider the system of 3 bodies equivalent to One body of Mass $M = m_1 + m_2 + m_3$ ~~at a position~~ Concentrated at the center-of-Mass position

Center-of-Mass coordinates:

Mass weighted Average position of a system of particles

consider N masses

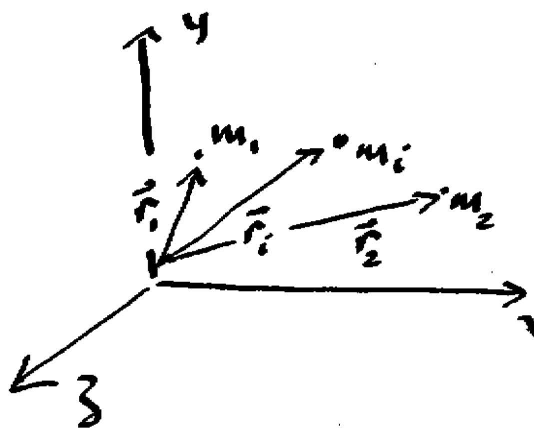
$$X_{c.m.} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_N x_N}{m_1 + m_2 + \cdots + m_N} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$$

$$Y_{c.m.} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$$

$$Z_{c.m.} = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$$

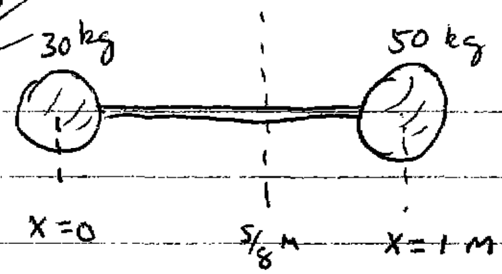
-or-

$$\vec{R}_{c.m.} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$



Easy to use for a system of discrete particles

Example

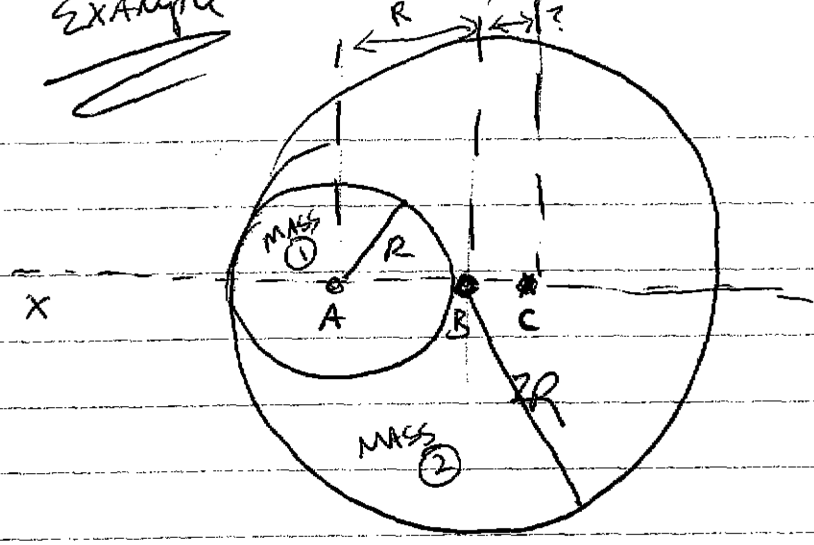


Where is the center of mass of the system

$$X_{cm} = \frac{30(kg)\,0\,m + 50(kg)(1\,m)}{(30+50)\,kg} = \frac{50}{80}\,m$$

$$= \frac{5}{8}\,m$$

EXAMPLE

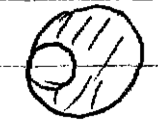


Circular metal plate w/ radius 2R

Circular plug of radius R is removed

Find The position of the center of Mass of the plate from which the disk was removed

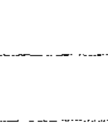
→ PT C in Drawing



X_B = Center of Mass of whole metal plate

~~C of M of disk ... C of M~~ can be thought of as made of two Masses

① circular disk w/ Mass centered at A

②  w/ Mass centered at C

$$X_B = \frac{M_1 X_A + M_2 X_C}{M_1 + M_2}$$

Let center of big plate be at $X=0$

$\Rightarrow X_B = 0$

$$X_C = -\frac{M_1 X_A}{M_2} = -\frac{[\pi R^2 (\rho t A)](-R)}{\pi (2R)^2 (\rho t A) - \pi R^2 (\rho t A)}$$

$\rho \equiv$ Mass density

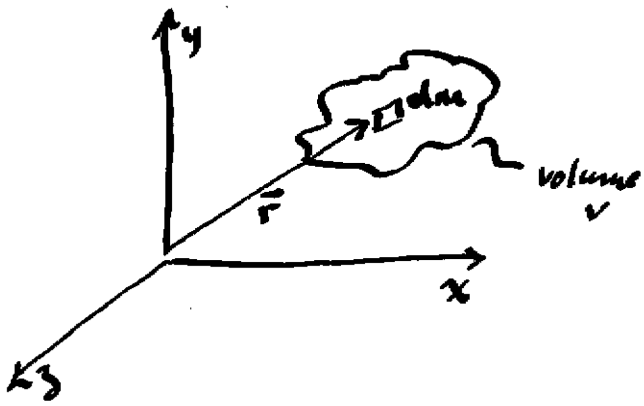
t = thickness

A = area

$(t)(A)$ = Volume

$$\boxed{X_C = \frac{1}{3} R}$$

Suppose we go to the continuous limit



$$x_{c.m.} = \int_V x\,dm \Big/ \int dm = \frac{\int_V x\,dm}{M}$$

$$y_{c.m.} = \int_V y\,dm \,/ M$$

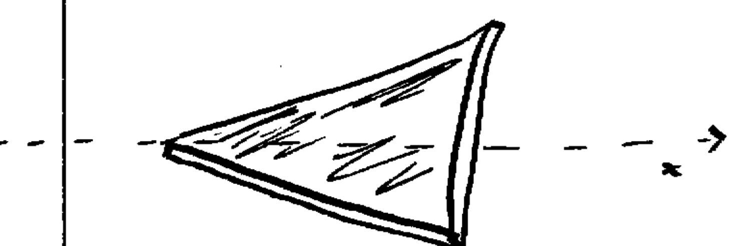
$$z_{c.m.} = \int_V z\,dm \,/ M$$

- or -

$$\vec{r}_{c.m.} = \frac{\int_V \vec{r}\,dm}{M}$$

What do I mean by center-of-mass?

What do I mean by center-of-weight?



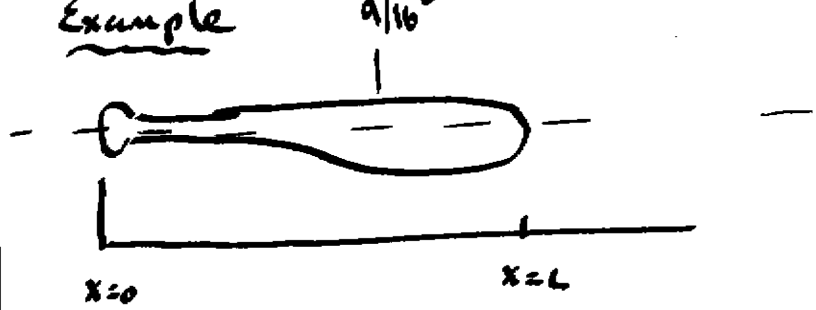
Suppose I had a triangular piece of metal (heavy). You had to hold it up in this orientation w/ one finger . . . where would you put your finger?

- or -

baseball bat

Discuss how "Mass weighting" allows one to find such a position

Example



Suppose a bat has a length L and a Mass/unit length

$$\lambda \equiv \lambda_0 \left(1 + \frac{x^2}{L^2}\right) \qquad 0 \le x \le L$$

(λ larger by factor of two at the thick end of the bat)
Find the c.o.m of the bat as a fn of L.

by symmetry ... lay bat along x ... can ignore y, z

-or- $y_{c.m.} = 0$
$z_{c.m.} = 0$

$$M = \int dm = \int_{x=0}^{x=L} \lambda dx$$

$$M = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) dx = \lambda_0 \left(x + \frac{x^3}{3L^2}\right)\Big|_0^L$$

$$M = \lambda_0 \left(L + \frac{L}{3}\right) = \frac{4L}{3} \lambda_0$$

$$x_{c.m.} = \frac{\int x dm}{\int M} = \frac{1}{M} \int x dm = \frac{1}{M} \int_0^L \lambda dx$$

$$x_{c.m.} = \frac{\lambda_0}{M} \left(\frac{x^2}{2} + \frac{x^4}{4L^2}\right)\Big|_0^L = \frac{\lambda_0}{M} \left(\frac{L^2}{2} + \frac{L^2}{4}\right) = \frac{\lambda_0}{M} \frac{3}{4} L^2$$

$$x_{c.M.} = \lambda_0 \frac{3}{4} L^2 \frac{3}{L 4 \lambda_0} = \frac{9}{16} L$$

back to a discrete system of Particles

$$M\vec{V}_{c.m.} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n$$

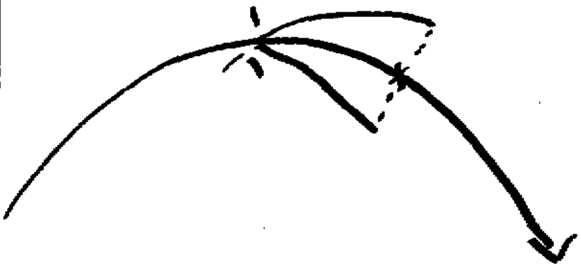
$$M\frac{d\vec{v}}{dt} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + \cdots + m_n\frac{d\vec{v}_n}{dt}$$

$$M\vec{a}_{c.m.} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_{n}$$

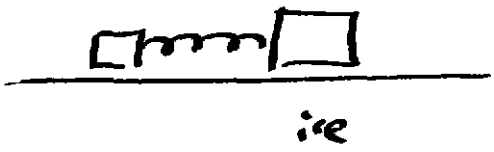
$$\parallel$$

$$\Sigma\vec{F} = \Sigma\vec{F}_{ext} + \underbrace{\Sigma\vec{F}_{int}}_{=0}$$

When a body or collection of particles is acted on by external forces, the center of mass moves just as though all the masses were concentrated at that point and it were acted on by a net force equal to the sum of external forces



Exploding projectile Example



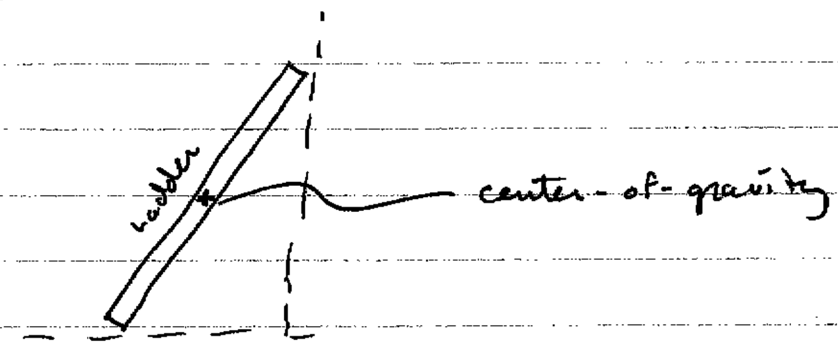
Masses on ice Example

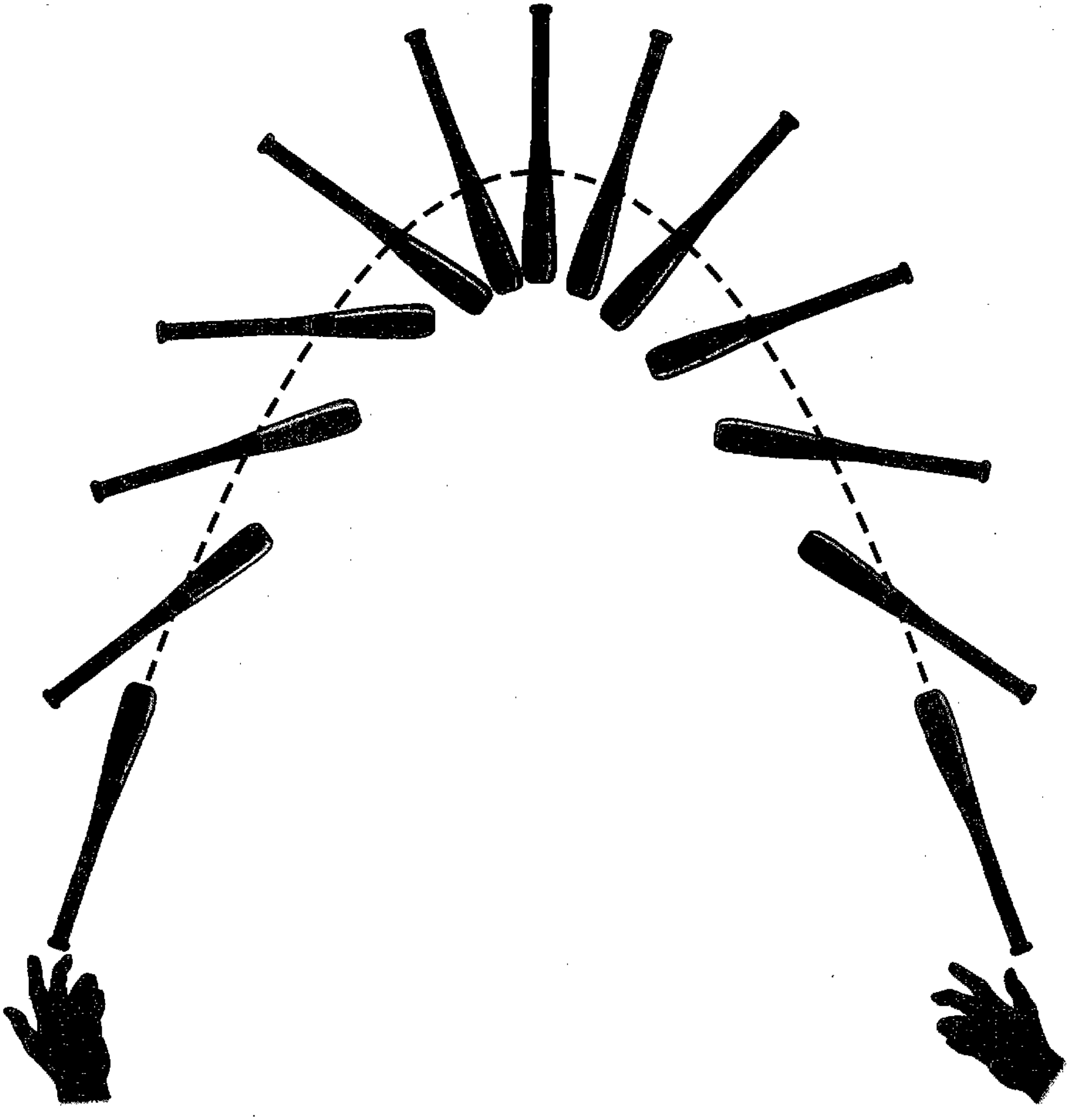
Center-of-Mass is a concept valid under any circumstance

Center-of-gravity - Point on a body where one can assume all mass is concentrated ~~for~~ ~~considering to~~ in considering how the force of gravity acts on a body

These points are one and the same.

Critical for rotational equilibrium





(b)