Theoretical ~~Che~~ Chemistry example

Vibration (IR spectra usually)

1 big, one small ← simplifies problem

Diatomic Molecule

$$F = -C/r^2 + D/r^3$$

What is frequency + Period of vibration

Equilibrium Position

$$F = 0$$

$$\frac{C}{r^2} = \frac{D}{r^3} \Rightarrow \text{ when } r = r_0 = \frac{D}{C}$$

Consider small oscillations about r_0 $\quad r = r_0 + x$

$$F = -\frac{C}{(r_0+x)^2} + \frac{D}{(r_0+x)^3} =$$

Taylor's Series expansion ... $\displaystyle -\frac{C}{r_0^2\left(1+\frac{x}{r_0}\right)^2} + \frac{D}{r_0^3\left(1+\frac{x}{r_0}\right)^3}$

$$(1+x)^{-2} = 1 - 2x + 3x^2 \dots$$

$$(1+x)^{-3} = 1 - 3x + 6x^2 \dots$$

Let $x = \frac{r \cancel{-r_0}}{r_0}$ $\quad$ x is small $\quad$ ignore terms $O(r/r_0)^2$

$$F = -\frac{C}{r_0^2(1+x)^2} + \frac{D}{r_0^3(1+x)^3}$$

$$F(x) = -\frac{C}{r_0^2}\left(1 - 2\frac{x}{r_0}\right) + \frac{D}{r_0^3}\left(1 - 3\frac{x}{r_0}\right)$$

$$= \underbrace{-\frac{C}{r_0^2} + \frac{D}{r_0^3}}_{0} + C\,2\frac{x}{r_0} - D\,\frac{3x}{r_0}$$

$$F(x) = m\frac{d^2x}{dt^2} = -\frac{1}{r_0}(3D - 2C)\,x$$

$$\frac{d^2x}{dt^2} = -\frac{1}{r_0 m}(3D - 2C)\,x$$

$$r_0 = D/C$$

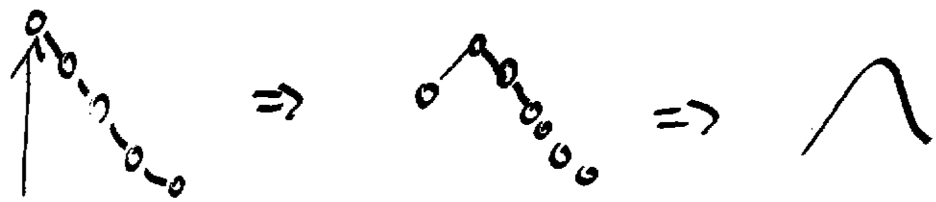
$$\frac{d^2x}{dt^2} = -\frac{C}{Dm}(3D - 2C)\,x$$

SHM w/ $\omega = \sqrt{\frac{C}{DM}(3D - 2C)}$

period $= \frac{2\pi}{\omega}$

WAVES

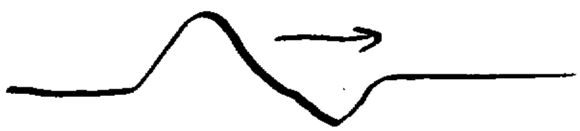
Consider SHM when adjoining particles are affected by the motion



etc....

The "Disturbance" propagates in The "Medium"
This is known as a Mechanical Wave.

The disturbance does NOT have to be SHM in nature
⇒ Single Transverse pulse on Slinky

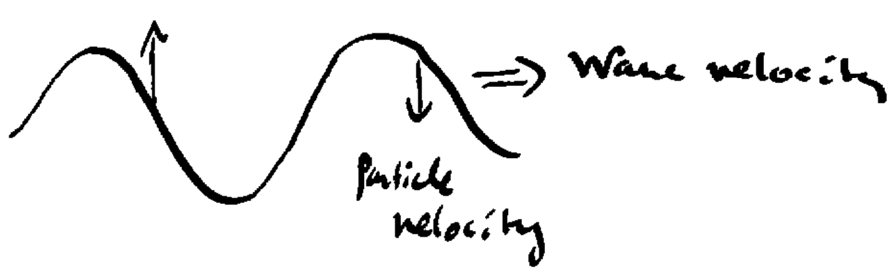


[Pulse vs. Continuous or Periodic wave] (in medium)

The motion of disturbed particles can be either transverse or longitudinal

⇒ longitudinal wave on Slinky

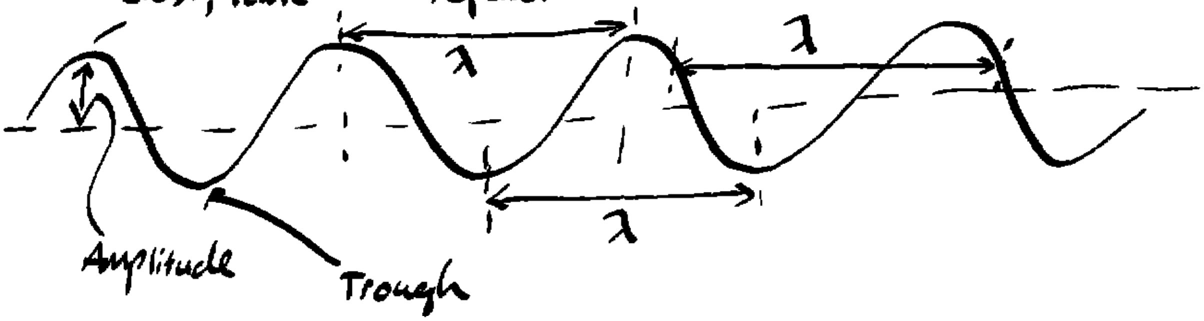
The wave disturbance Travels



Perturbation done only at one frequency

Consider a Periodic harmonic Wave

disturbance repeats



$\lambda \equiv$ wavelength (peak to peak distance, for example)

$T \equiv$ Period Time (in s) for each peak to move to the old position of the next peak so that the wave looks the same. Or time for disturbance at one position to go through one complete cycle of its motion.

Consider peaks moving to right

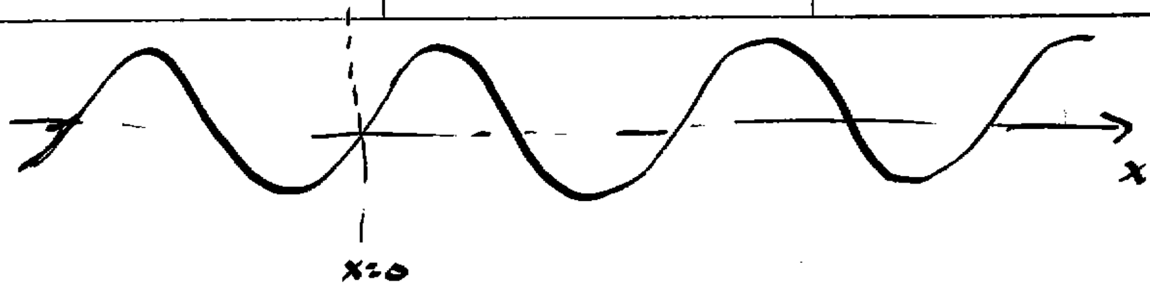
$$\text{Velocity of wave} \equiv v = \frac{\lambda}{T}$$

$$\text{frequency} = \frac{1}{T} \quad \text{(just like in SHM)}$$

$$\equiv \nu$$

$$\therefore \boxed{v = \lambda \nu}$$

Lets freeze the wave at a given time (say $t=0$)
(wave already been propagating for a while)



$y(x) = A \sin(kx)$

where A gives the Amplitude

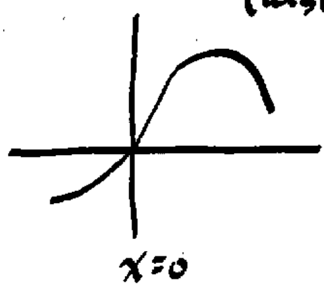
$k = \frac{2\pi}{\lambda}$ because if $x = \lambda$ or 2λ or $n\lambda$

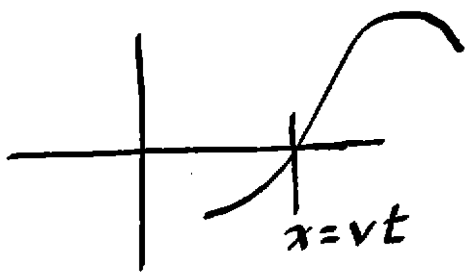
The fn must look the same

$k \equiv$ Wave Number

let's let the wave travel for a time t

Pulse





time = t

$y = f(x)$

$y = f(x - vt)$

So – Mathematically we can take a "Frozen" (in time) equation of a wave and Make it travel to the right by letting $x \to x - vt$

Similarly can Make it go to the left w/ $x \to x + vt$

let's Make our Frozen periodic wave above travel to the Right

$$y(x) = A \sin[k(x - vt)]$$
$$= A \sin(kx - kvt)$$

$$kvt = \frac{2\pi}{\lambda} v t = 2\pi\nu t$$

$$\frac{v}{\lambda} = \nu \qquad 2\pi\nu = \omega = \frac{2\pi}{T}$$

$V = \lambda \nu$ just as in SHM

So

Periodic wave moving to the right

$$y(x,t) = A \sin(kx - \omega t)$$

two variables now!

Could have arbitrary starting phase

use either Sin or Cosine

general eqn for Harmonic Wave —

$$y(x,t) = A \sin(kx - \omega t + \phi)$$

$$[\text{or} \quad A \cos(kx - \omega t + \phi)]$$

Transverse waves moving on a string are a useful example for demonstrating wave characteristics

What properties of a wave depend on the Medium??

Amplitude?

λ?

ν?

Velocity?

Know about medium should be able to calc. properties of wave

$$F\frac{\Delta l}{R} = \mu\frac{\Delta l}{R} v^2$$

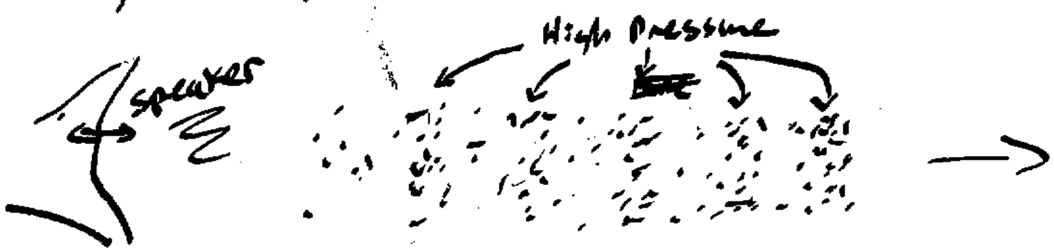
$$\sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}}$$

or $v = \sqrt{\frac{F}{\mu}}$ or $\sqrt{\frac{T}{\mu}}$ ~ Tension

heavier rope → v smaller
larger Tension → v faster

for <u>longitudinal</u> <u>waves</u> have a similar thing
Such as Sound waves

longitudinal pressure variations in medium



$$v_{\text{longit. wave in liquid or gas}} = \sqrt{\frac{B}{\rho}}$$ ~ Bulk Modulus

just a constant for given Medium
We did NOT cover → p. 342 of text

B ~ Tells how compressible a gas, fluid is
for ~~a~~ a given pressure

wave on a string $V = \sqrt{\dfrac{\text{Tension}}{\mu = \text{mass/length}}}$

sound waves

$V = \sqrt{\dfrac{B}{\rho}}$ — Bulk modulus

defines the compressibility of medium

ρ: volume density

Transverse wave

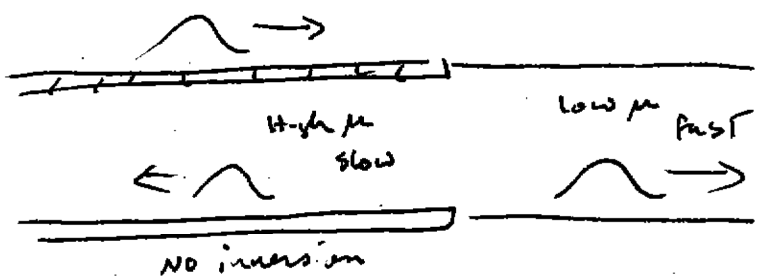
Fixed end

Phase change of 180° at reflection

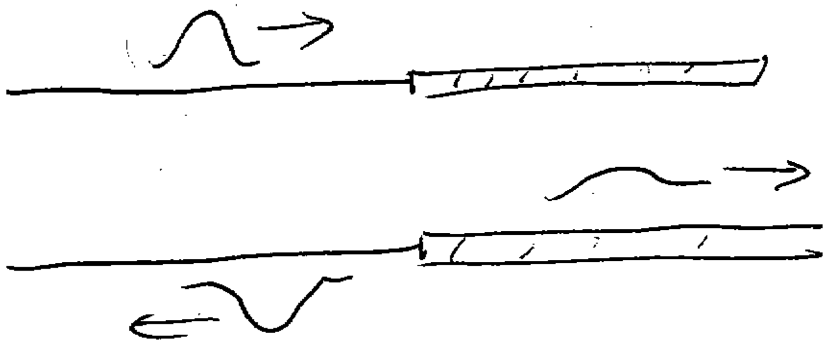
loose end

No phase change at reflection

Interface



Slow to fast reflected wave has NO inversion

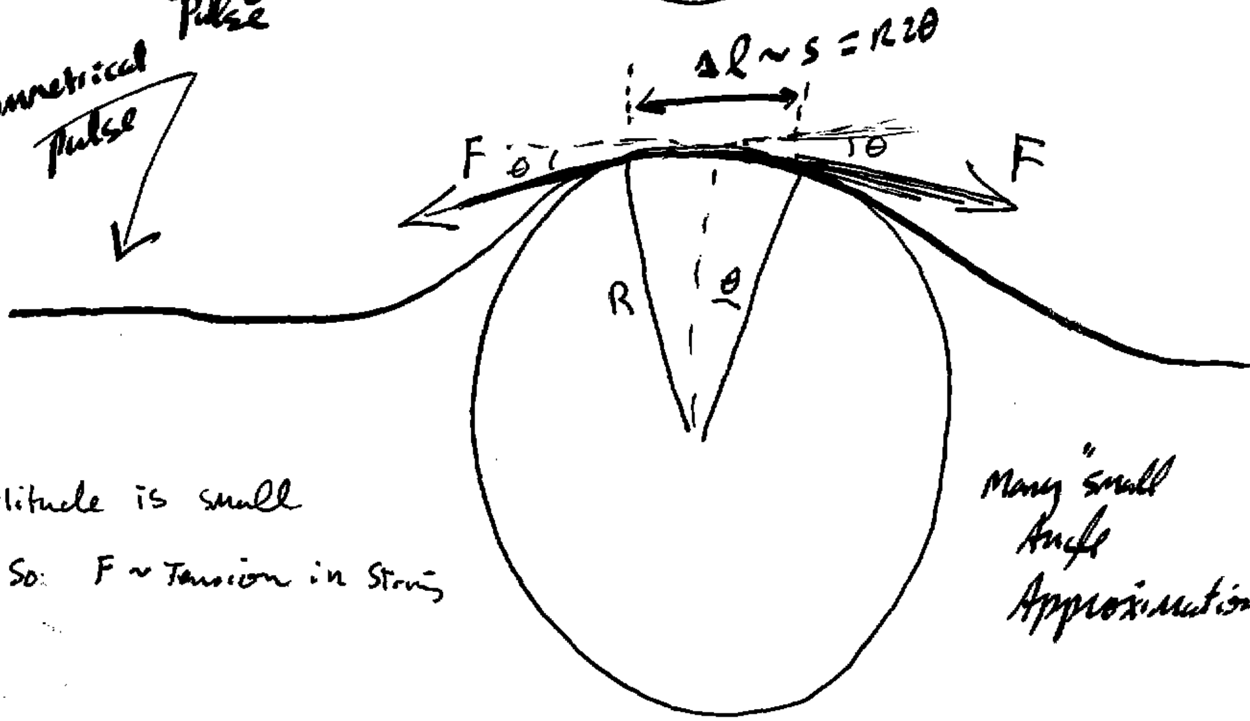


fast to slow reflected wave has inversion

Calculate velocity of wave on a string w/ Tension ~~T~~ $\equiv F$

To avoid confusion w/ Period

and Mass/length $\equiv \mu$



Amplitude is small

So: $F \sim$ Tension in String

Many "small Angle Approximations"

Horizontal components of 2 F vectors cancel

Vertical restoring force $\sim 2F\sin\theta \sim 2F\theta \sim F\dfrac{\Delta l}{R}$

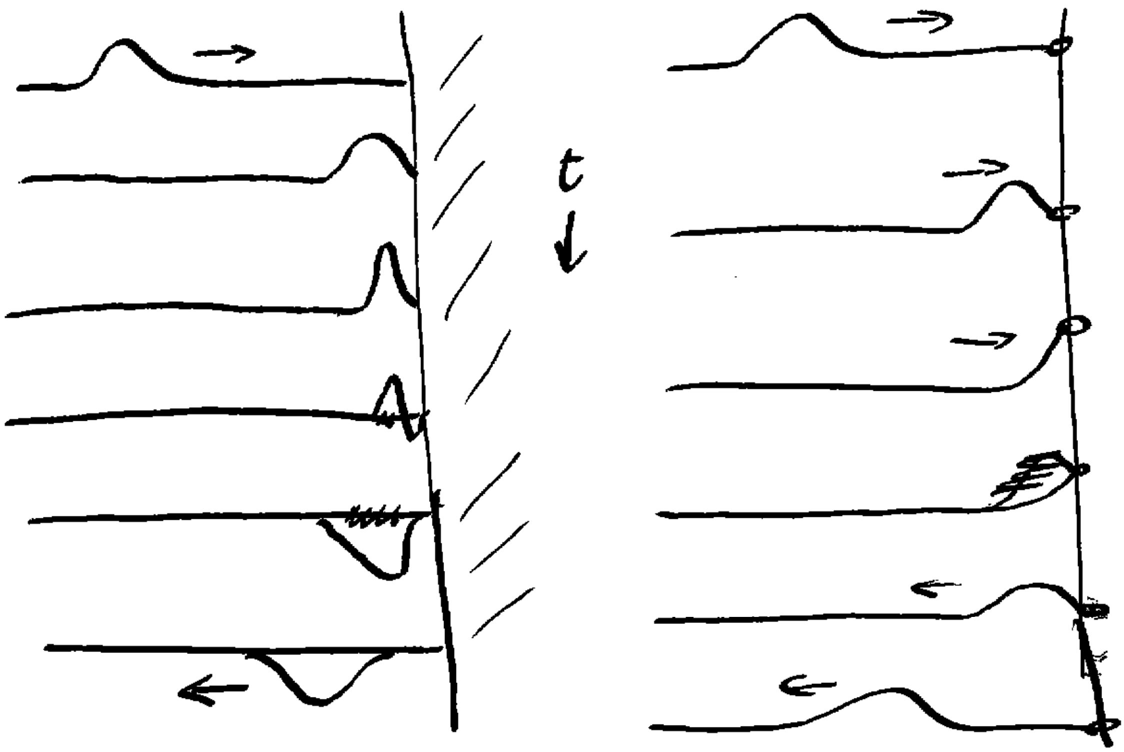
Mass of string segment $\Delta m = \mu\Delta l$

Mass segment is moving on a circle for the moment

$\therefore \quad m\dfrac{v^2}{r} =$ vertical force

Waves hitting obstacles

All reflected ... what is phase of reflected wave?



Fixed end

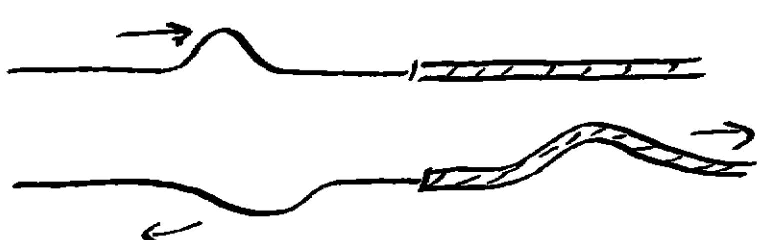
→ phase change of 180° at reflection

→ phase inversion

loose end

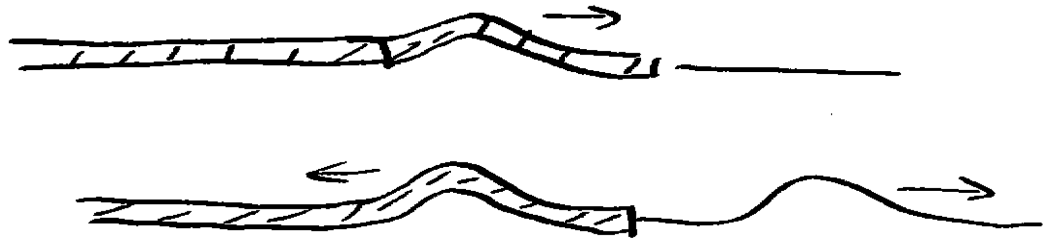
→ NO phase change at reflection

Waves encountering an interface between Media W/ different characteristics have reflected and transmitted components



Wave incident on boundary

Travelling from "fast" to "slow" Medium



"slow to fast" media

Transmitted wave ... both cases → No phase inversion

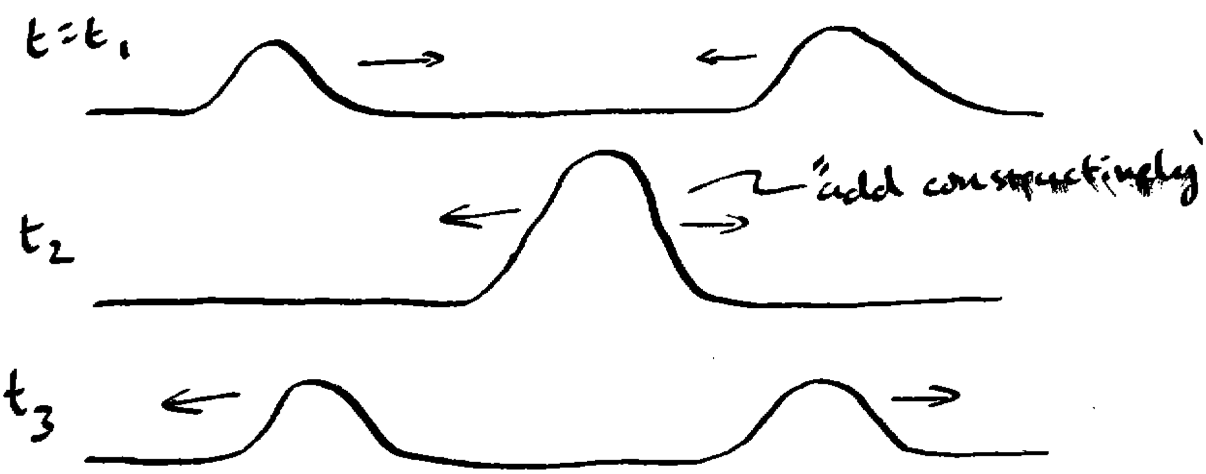
<u>Reflected wave</u> $\begin{cases} \text{fast to slow} \\ \text{low } \mu \text{ to high } \mu \end{cases}$ → 180° phase inversion

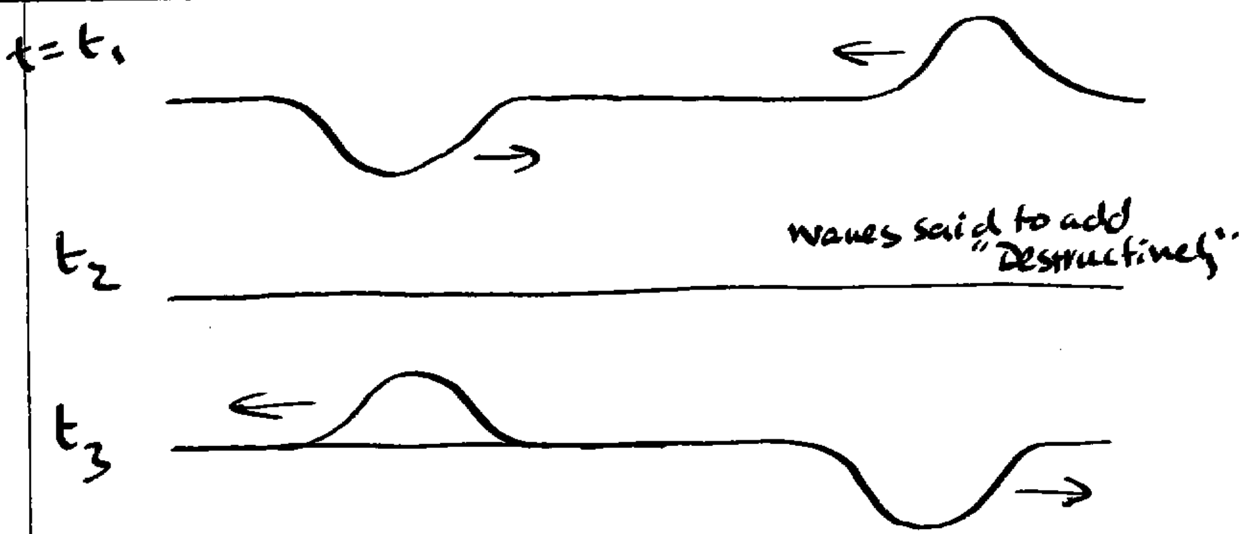
$\begin{cases} \text{slow to fast} \\ \text{high } \mu \text{ to low } \mu \end{cases}$ → No phase inversion

What happens when two waves meet one another

<u>Principle of Superposition</u>

When two or more waves pass thru a given point simultaneously the resultant displacement is the sum of the individual displacements





Two periodic waves superimposed on one another in some way are said to <u>Interfere</u>.

⇒ This is an incredibly rich + useful phenomenon!

Music
Antireflective coatings
best techniques for measuring small distances
Allows us to split light into colors
etc.

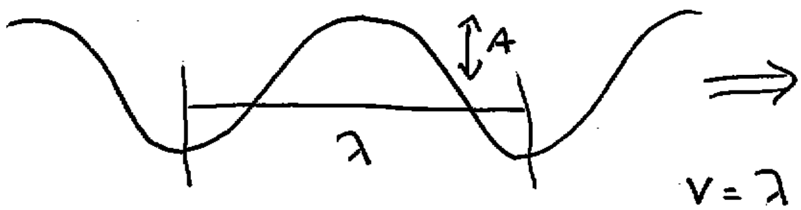
Consider 2 Harmonic waves w/ Amplitude A and frequency ω
Traveling opposite directions in a string
→ one being the reflection of the other!

We've learned that two waves are out of phase by 180° due to the "boundary condition" of the fixed end at the wall

Nov 30

Have Students do wave
Arbitrary SHO
timed SHO

Waves



$$v = \lambda \nu = \lambda \frac{1}{T}$$

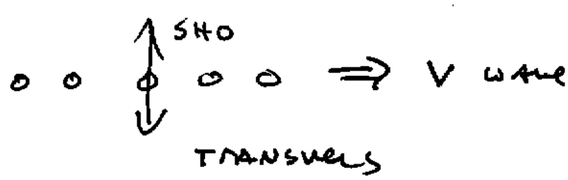
Moves to right

$$y(x,t) = A \sin(kx - \omega t + \varphi) \quad \underline{\underline{or}} \quad A \cos(kx - \omega t + \varphi)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Moves to left

$$y(x,t) = A \sin(kx + \omega t + \varphi) \quad \text{or} \quad \text{'' } +$$



Longitudinal