Work + Energy

What is energy ?

What is Work ?

Push a car for 1 mile ... is that work? ... yes

(Force)(distance) ~ Work

We'll define more carefully soon

Energy ... ~~loosely~~ loosely ... Ability to do Work.

Energy is Conserved. That is to say ~~thing~~ cannot be ~~neither~~ created OR destroyed!

Ask ... can energy be created or destroyed?

IT can take on Many different forms

Mechanical Energy

Mass energy - $E = mc^2$

heat energy

Kinetic energy - Energy associated w/ Motion

Potential energy - Energy stored in system due to configuration

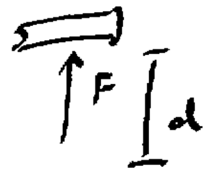
Work, Energy ... different faces of the same thing

Same units $\rightarrow$ MKS Joule

$$\text{Joule} \sim (\text{force})(\text{distance}) = N\,m = kg\frac{m}{s^2}\,m = kg\frac{m^2}{s^2}$$

Consider book ... Pick it up

do work against gravity

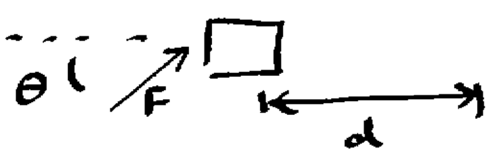


Suppose I carry it around
o'ing height
Is this work?

It may seem so ... but in physics we say NO.
You're NOT ~~enhanced~~ changed ability of book to do work
i.e. changed its energy.
whereas you did when you picked it up

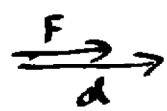
Work ≡ The work done by a force on a particle (or object) is equal to the force times the distance through which that particle (or object) is moved along the line of the force

work = Fd

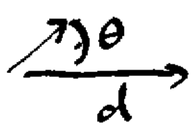


Work = $F\cos\theta\, d$

$\vec{d}$ is a vector ... displacement



work = Fd



work = $F\cos\theta\, d$

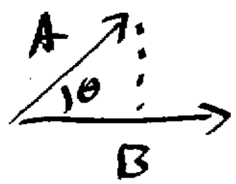
or

$Fd\cos\theta$

Work ≡ component of force along direction of movement times the distance moved

-or-

≡ Force times component of distance moved along the direction of force.



2 vectors w/ angle between

$$|\vec{A}|_{\text{along } B} = A\cos\theta$$

$$|\vec{B}|_{\text{along } A} = B\cos\theta$$

use this very often ... let's define a vector Dot product

$$\underbrace{\vec{A}\cdot\vec{B}} = |\vec{A}||\vec{B}|\cos\theta$$

gives the (projection of $\vec{A}$ along $\vec{B}$) x ($|\vec{B}|$)

or ($|\vec{A}|$) x (projection of $\vec{B}$ along $\vec{A}$)

Also

$$\vec{A}\cdot\vec{B} \; \cancel{\text{also}} = A_x B_x + A_y B_y + A_z B_z$$

NOTE these are Scalars, NOT vectors

Dot product also called Scalar product

Proved on p 19,20 of text

$$\vec{A}\cdot\vec{A} = ?$$

$$= |\vec{A}||\vec{A}|\cos\theta = |\vec{A}||\vec{A}|$$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos 90 = 0$

No projection possible here!

Example

$\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{B} = 1\hat{i} + 0\hat{j} + 9\hat{k}$

Ask for Arbitrary (bet 0 + 10) components from class

$\vec{A} \cdot \vec{B} = (3)(1) + (4)(0) + (2)(9) = 4 + 18 = 22$

What is opening angle between $\vec{A}$ and $\vec{B}$?

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$|\vec{A}| = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{29}$

~~22 =~~

$|\vec{B}| = \sqrt{82}$

$22 = \sqrt{29}\sqrt{82} \cos\theta$

Solve for θ $\qquad \theta \sim 63°$

Since Work uses the projected component of Force along distance $\Rightarrow$ conveniently defined in terms of vector scalar or dot product

$W \equiv \vec{F} \cdot \vec{S}$ for constant vectors

$dw = \vec{F} \cdot d\vec{s}$

$W = \int \vec{F} \cdot d\vec{s}$ for varying ~~force~~ force / path

$W = \int (F_x dx + F_y dy + F_z dz)$

NOTE $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0 = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = (1)(1)\cos 90^\circ = 0$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

EXPAND

$$= A_x\hat{i}\cdot B_x\hat{i} + A_x\hat{i}\cdot B_y\hat{j} + A_x\hat{i} B_z\hat{k}$$

$$+ A_y\hat{j}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + A_y\hat{j}\cdot B_z\hat{k}$$

$$+ A_z\hat{k}\cdot B_x\hat{i} + A_z\hat{k}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k}$$

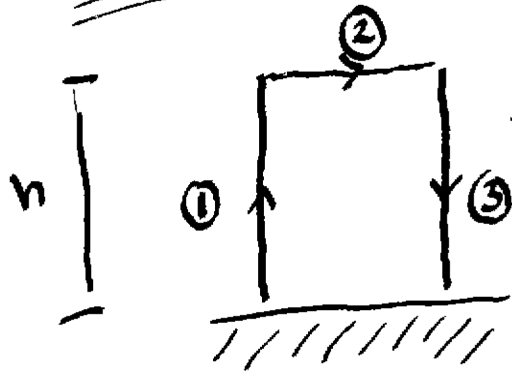
$$= A_x B_x\,\hat{i}\cdot\hat{i} + A_x B_y\,\hat{i}\cdot\hat{j} + A_x B_z\,\hat{i}\cdot\hat{k}$$

$$+ \ldots$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Scalar or dot product in terms of components

Example



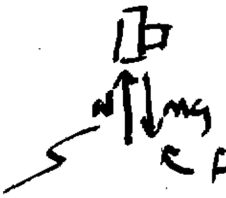
Rock climber's path on a cliff

climber carries a 35 mm camera. How much work does the climber perform to lug camera on trip.

Assume Movement at Constant Velocity $\rightarrow a = 0$

$\Rightarrow$ hell of a good rock climber!!

Free body diagram

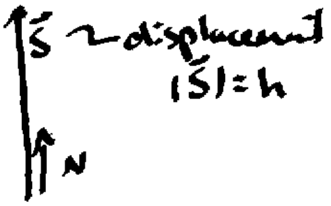


$a_y = 0 \therefore N = mg$

force of gravity on camera

Force rock climber exerts to support camera

(1)

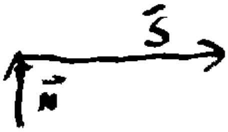


$$\text{Work}_{(1)} = \vec{N}\cdot\vec{S} = |\vec{N}||\vec{S}|\overbrace{\cos\theta}^{+1} = mgh$$

$|\vec{N}| = mg$

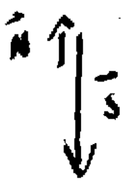
$|\vec{S}| = h$

(2)



$$\text{Work}_{(2)} = \vec{N}\cdot\vec{S} = |\vec{N}||\vec{S}|\overbrace{\cos\theta}^{0} = 0$$

(3)



$$\text{Work}_{(3)} = \vec{N}\cdot\vec{S} = |\vec{N}||\vec{S}|\overbrace{\cos\theta}^{-1} = -mgh$$

Net TOTAL work = $\text{Work}_{(1)} + \text{Work}_{(2)} + \text{Work}_{(3)} = 0$ Joules

Negative Work! ... don't you wish you could get a job doing that!

What work does gravity do in part (1)?

① On climb up ... there is NET work
where did it go?

Lift camera up a distance h ... can let it go ... it can fall and can do work! ... So by lifting camera up you've increased its energy ≡ ability to do work

We say you have increased its "Potential Energy"

Converted chemical energy in body to perform work on camera to increase its potential energy.

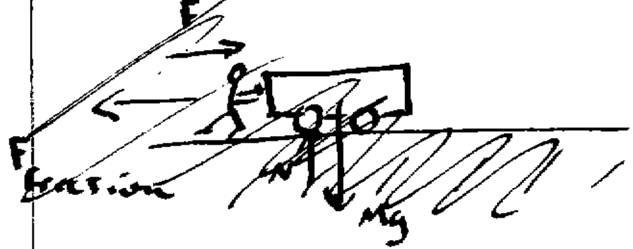
②

NOT increasing or decreasing Potential energy ... NO work

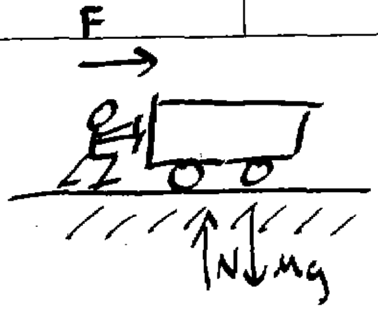
③

decreasing potential energy
Negative Work

NET work = Net potential energy change = 0



You push cart with
~~Have to~~
F



Push a frictionless cart on a frictionless horizontal Surface.

Net horizontal force $F = M_{cart} a_{cart}$

Say you push w/ const force F over Distance D

$$W = FD$$

You put work into the system ... where has energy gone.
You've NOT changed the Potential Energy of the system.

However $V^2 = V_0^2 + 2a(x-x_0)$

$$V^2 = 2\frac{F}{M}D$$

$$\Rightarrow \underbrace{\tfrac{1}{2}mv^2}_{\text{Kinetic Energy}} = FD \leftarrow \text{work input}$$

The work is converted into "Kinetic energy" — Energy of Motion.

Kinetic Energy of an object mass m, moving w/ velocity of magnitude $|\vec{V}|$ $\equiv \frac{1}{2}mv^2$ / Important + General