Lect Sept 10

Last class

$$V_{average} = \frac{\Delta x}{\Delta t}$$

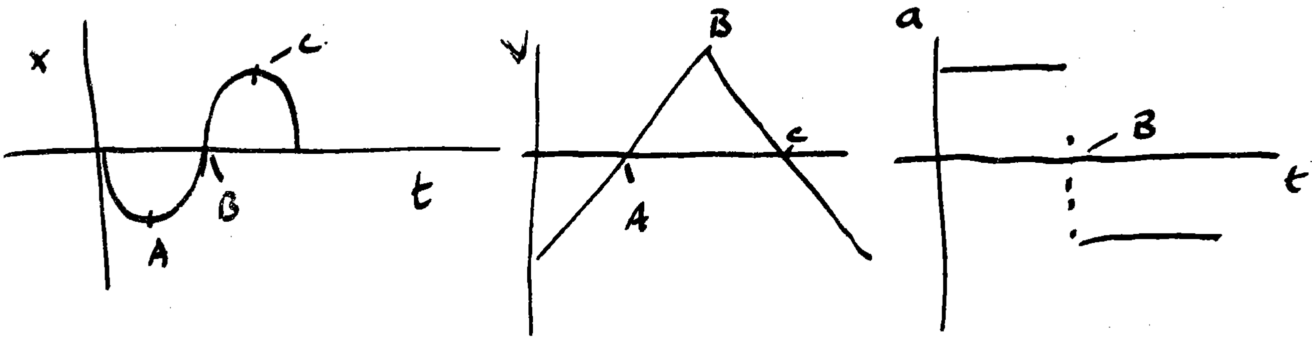
$$V_{instantaneous} = \frac{dx}{dt}$$

$$a_{average} = \frac{\Delta v}{\Delta t}$$

$$a_{instantaneous} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$x, a, v, t \rightsquigarrow$ Kinematic Variables

These Are <u>NOT</u> independent Variables



Started an example

$$x(t) = At^2 + Bt^3$$

$$x(t) = \left(3\ \tfrac{m}{s^2}\right)t^2 + \left(0.3\ \tfrac{m}{s^3}\right)t^3$$

$$\cancel{V(t) = 6t + (.3)(3)}$$

$$V(t) = 2At + 3Bt^2$$

$$a(t) = 2A + 6Bt$$

what about v ? and a ?

$$x(t) = A\cancel{t}^2 - Bt^3$$

$$v = \frac{dx}{dt} = 2At - 3Bt^2$$

$$a = \frac{d^2x}{dt^2} = 2A - 6Bt$$

Fill in Table

t (s)	x (m)	v (m/s)	a (m/s²)
0	0	0	6
1	2.7	5.1	4.2
2	9.6	8.4	2.4
3	18.9	9.9	.6
4	28.8	9.6	-1.2

NOTE: I don't say 9.60000003 m

Significant Figures

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{9.6}{4} = 2.4 \ m/s$$ in + ~~x~~ direction

graph x vs t

v vs t

a vs t

$\cancel{X(t) = 10t^2 + 1t^3}$

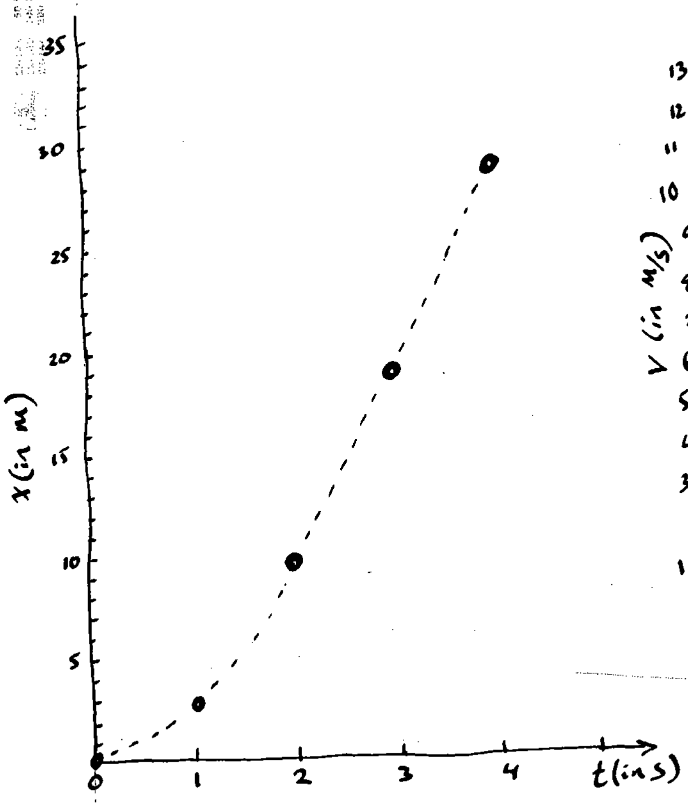
Position of Car as Fn of Time

$$X(t) = 3t^2 - 0.3t^3$$

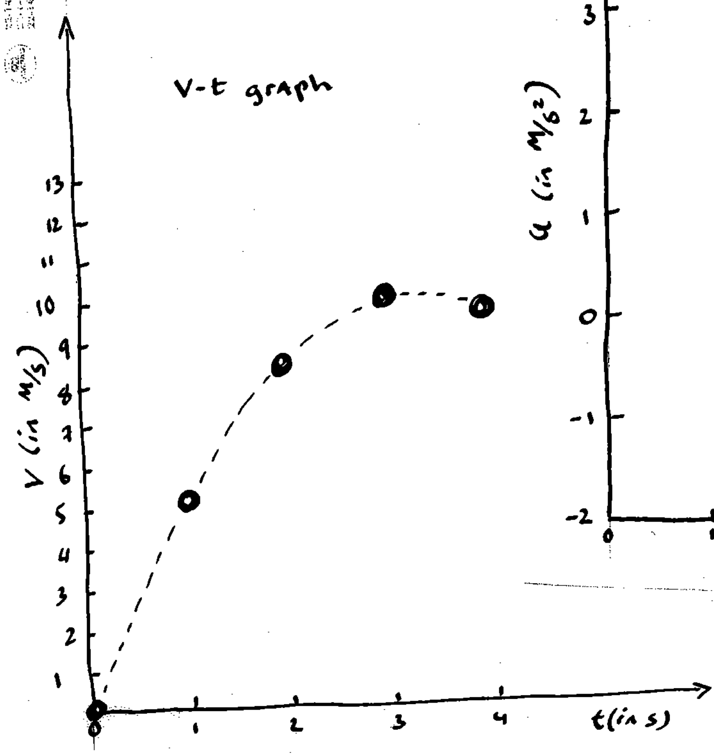
$$V = \frac{dX(t)}{dt}$$

$$a = \frac{dV(t)}{dt} = \frac{d^2X(t)}{dx^2}$$

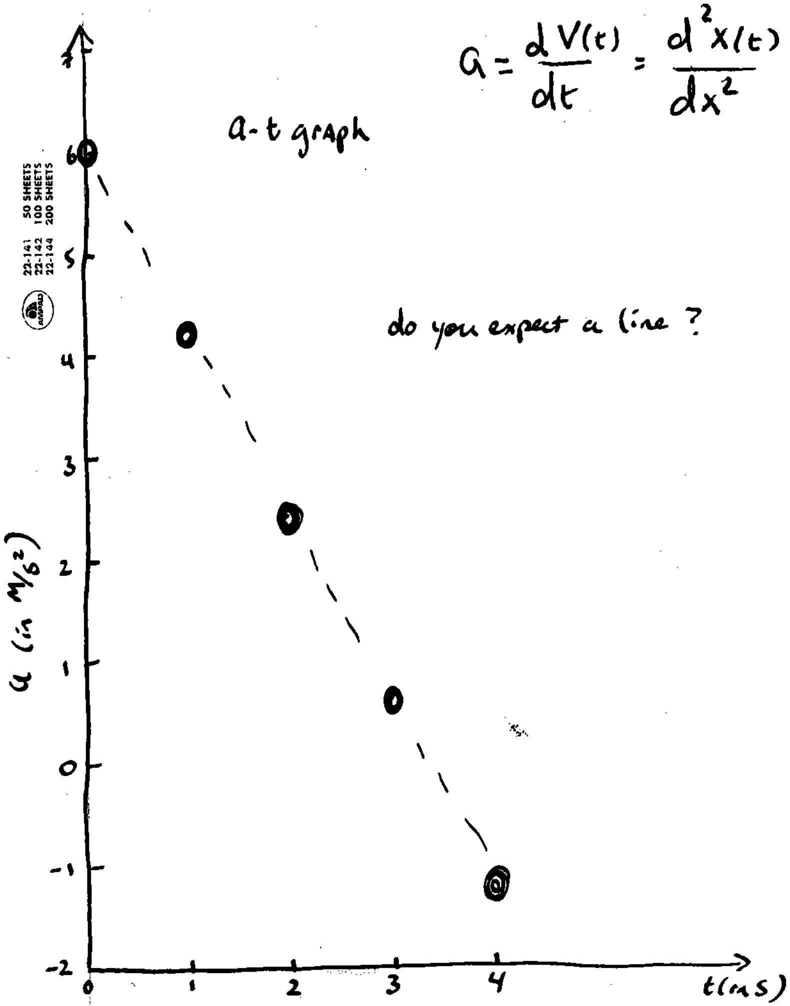
x-t graph



v-t graph



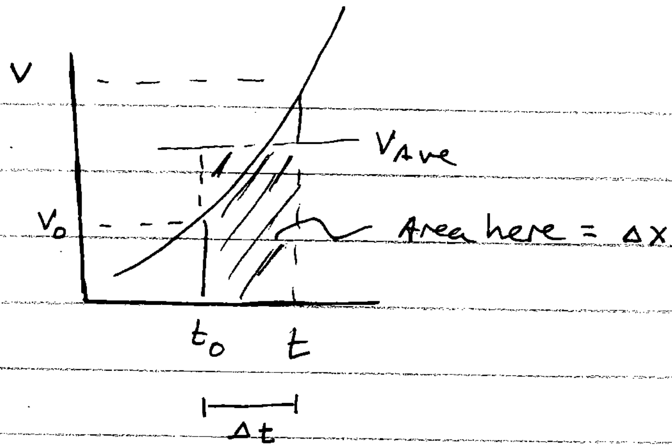
a-t graph



do you expect a line ?

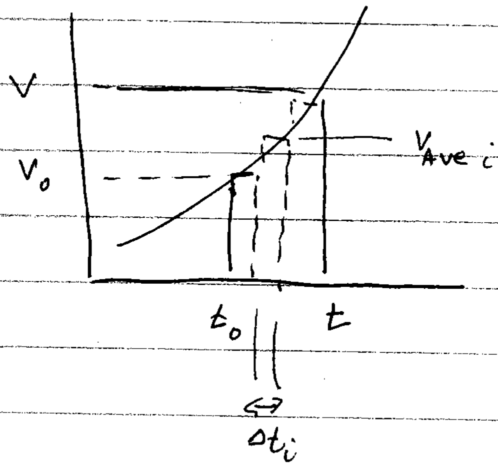
$$\frac{\Delta x}{\Delta t} = V_{Ave}$$

$$\Delta x = V_{Ave}\,\Delta t$$



Could Try to be More Accurate

$$\Delta x = \sum_i V_{Ave_i}\,\Delta t_i$$



In The limit $\Delta t_i \to 0$

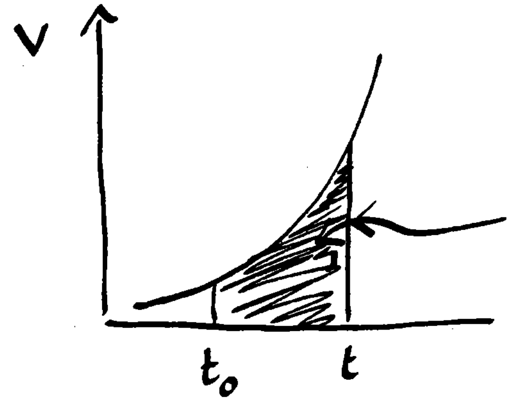
$$\Delta x = \lim_{\Delta t_i \to 0} \sum_i (V_{Ave})_i\,\Delta t_i$$

$$\left.\begin{array}{l} \Delta t \to dt \\ V_{Ave} \to V_{INSTANTANEOUS} \end{array}\right\} \text{ in this limit}$$

$$\Delta x = \int_{t_0}^{t} v\,dt$$

In this limit

Δx = exact area under v-t curve between limits of $t = t_0$ and $t = t$



area under curve = Δx

Should this surprise you?

recall from calculus

If $g = \int f\,dt$, then $f = \dfrac{dg}{dt}$

Integrals + derivatives are $\approx$ inverses of each other

↳ Think of as a slope

↳ Think of as a sum

$\left.\begin{array}{l} dx \\ dt \\ dv \\ \vdots \end{array}\right\}$ $\equiv$ differentials ... Think of as a little tiny Δx ....

* give table of integration formulas

Why are CONSTANT Acceleration problems encountered so frequently ?

Gravity

gravitational Mass

$$F = \frac{G M_1 M_2}{r^2} \qquad F = \frac{G M M_E}{R_E^2}$$

$$F = ma \Rightarrow a = \frac{G M_E}{R_E^2} \sim g \sim \text{CONSTANT}$$

Inertial Mass

Gravitational vs. inertial Mass

- bubble gum
- chocolate
- Apple
- Cantelope

$$\Delta x = \int_{t_0}^{t} v\,dt$$

example

$V = 3t + 4$

$x - x_0 = \int_{t_0}^{t} (3t+4)\,dt$

$= \left[\frac{3t^2}{2} + 4t\right]_{t_0}^{t}$

$x - x_0 = \frac{3t^2}{2} + 4t - \frac{3t_0^2}{2} - 4t_0$

$$\boxed{x - x_0 = \int_{t_0}^{t} v\,dt}$$

Very general eqn

V can vary w/ t

Can evaluate if $V(t)$ is known

$$\frac{\Delta v}{\Delta t} = a_{ave}$$

↳ do same steps we just did ...

$$\boxed{v - v_0 = \int_{t_0}^{t} a\,dt}$$

Very general eqn

a can vary w/ t

can evaluate if

$a(t)$ is known

Now

Consider special case of a constant w/ time

→ Constant Acceleration problem ⇒ Very IMPORTANT

$$v - v_0 = \int_{t_0}^{t} a\,dt = a\int_{t_0}^{t} dt = a(t - t_0)$$

- gravity on surf of earth
- Any constant "force" situation

Usually define $t_0 = 0$

$$\boxed{v = v_0 + at}$$

uses v, a, t NOT x

Substitute this into general eqn for x

$$x - x_0 = \int_{t_0}^{t} (v_0 + at)\,dt$$

NOTE:

$$\int (A + B)\,dt = \int A\,dt + \int B\,dt$$

$$x - x_0 = \int_{t_0}^{t} v_0\,dt + \int_{t_0}^{t} at\,dt$$

$$x - x_0 = v_0 \int_{t_0}^{t} dt + a \int_{t_0}^{t} t\,dt$$

$$x - x_0 = v_0\, t \Big|_{t_0}^{t} + a\,\frac{t^2}{2}\Big|_{t_0}^{t}$$

$$x = x_0 + v_0 (t - t_0) + \frac{1}{2} a (t^2 - t_0^2)$$

If $t_0 = 0$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

uses x, t, a NOT v

End lect Sept 9

The game is To find eqns that use variables you have or need to find ... and NOT Variables you don't have

So far have ① eqn that does NOT ~~have~~ use x

① eqn that does NOT use v

Need ① eqn that does NOT use t

① eqn that does NOT use a

Consider $\Delta x = V_{Ave} \Delta t$

for case w/ CONSTANT Acceleration

$\Rightarrow$ V changes at a CONSTANT rate

$\therefore$

$$V_{Ave} = \frac{V + V_0}{2}$$

V final $\therefore$ at end of interval

$\therefore$

$$x - x_0 = \left(\frac{V + V_0}{2}\right) t$$

$$\boxed{x = x_0 + \left(\frac{V + V_0}{2}\right) t}$$

x, v, t No a

Finally find eqn w/ NO t

START w/ $v = v_0 + at$

Solve for t $\longrightarrow$ $t = \dfrac{V - V_0}{a}$

Subst. in above eqn

$$x = x_0 + \left(\frac{V + V_0}{2}\right)\left(\frac{V - V_0}{a}\right)$$

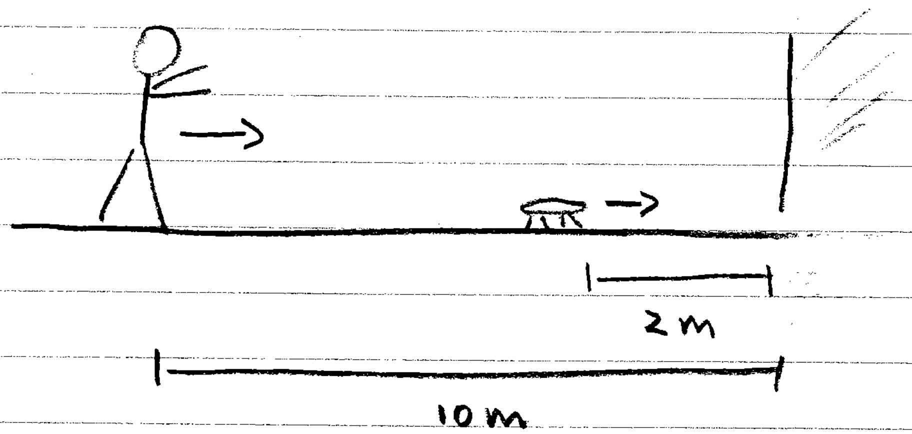
$$2a(x - x_0) = V^2 - V_0^2$$

$$\boxed{V^2 = V_0^2 + 2a(x - x_0)}$$

v, a, x NO t

CAN Tie 2 1d problems into one problem

example



you see cockroach at $t=0$
you start from rest and Accelerate (CONSTANT)
toward cockroach at $8\ m/s^2$
cockroach moves at const v of $1\ m/s$
does cockroach make it to safety?

Time for cockroach to make it safely
$a=0$ const

$$x_c - x_{0_c} = \left(\frac{v_{0_c} + v_{f_c}}{2}\right) t \qquad 2m = 1\ m/s\ t(s)$$

$$t = 2s$$

If person can reach wall in $t \leq 2s$
cockroach dies!

$$x_p = x_{0_p} + v_{0x_p} t + \frac{1}{2} a_{x_p} t^2$$

$$10\ m = 0t + \frac{1}{2} 8t^2 \qquad 10 = 4t^2 \qquad t = 1.6\ s < 2s$$