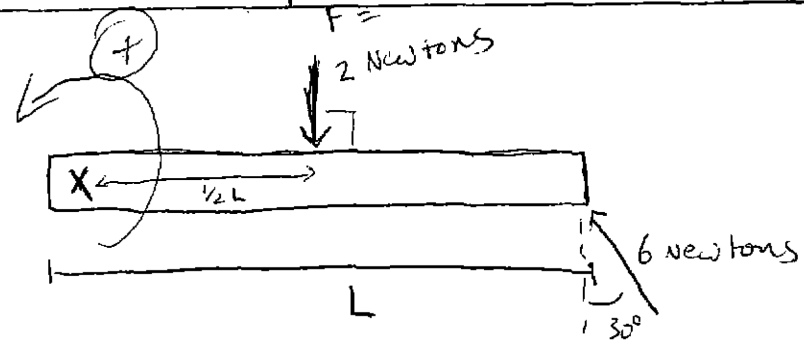
3/27/01

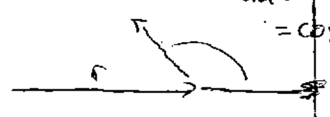


A rod of length, L, is free to rotate about ~~a point~~ an axis at one end. The forces shown act on the rod at a given instant. The mass of the rod is 3 kg, L = 2 m

Find $\vec{\alpha}$ of rod at this instant.

$$\Sigma \vec{\tau} = I \vec{\alpha}$$

$I = \frac{1}{3} M L^2$ from Table



Sin 120 = cos 30

$$\Sigma \vec{\tau} = \underbrace{-(2)(\tfrac{1}{2} L)}_{\text{(clockwise)}} + L\underbrace{\{6 \cos 30\}}_{\text{Component of F} \perp \text{to r}}$$

$$\Sigma \vec{\tau} = -2 + (2)(5.2) = 8.4 \ N \cdot M$$

$$\therefore \quad \underbrace{8.4 \,(N \cdot M)}_{kg\, M/s^2} = \underbrace{\tfrac{1}{3} M L^2}_{\frac{1}{3}(3)(2)} (kg\, m^2)\ \alpha$$

$$kg \frac{m^2}{s^2} = kg\, m^2 \qquad \alpha \sim \frac{1}{s^2}$$

$$\frac{8.4}{2} = \alpha = 4.2 \ {rad}/{s^2}$$ counterclockwise as you look

$$\vec{\alpha} = 4.3 \ {rad}/{s^2} \text{ out of paper}$$

Right Hand Rules ... Practice

what direction is $\vec{\omega}$?

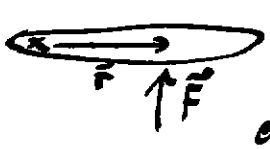
$\vec{\omega}$ out of paper

Suppose $\vec{\alpha}$ is increasing $|\vec{\omega}|$ ... what direction is $\vec{\alpha}$ ...

out of paper

Suppose $\vec{\alpha}$ is reducing $|\vec{\omega}|$ ... what direction is $\vec{\alpha}$

into paper

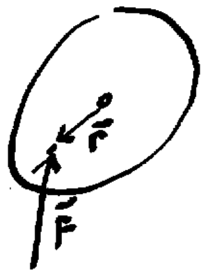


$\vec{\tau}$ is in what direction?

out of paper

$\vec{\tau} = \vec{r} \times \vec{F}$

0 Torque



into paper

Get good at these Right hand Rules !!

→ use to find $\vec{\omega}$, $\vec{\alpha}$ directions

→ use to find $\vec{C} = \vec{A} \times \vec{B}$ direction

→ use to find $\vec{\tau} = \vec{r} \times \vec{F}$ direction

We will do more w/ torque when we get to static equilibrium.

Angular momentum

What do we mean by Momentum?

$$\frac{dp}{dt} = F \qquad dp = F\,dt$$

To change momentum you must exert a force for a time.

if Momentum is larger ... must exert a larger force to stop an object in same time interval

what is Angular ~~analo~~ analogue of Momentum?

$$P = mV \qquad v = r\omega$$

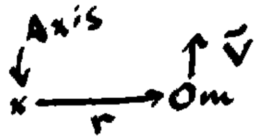
Angular Momentum $\sim I\omega$

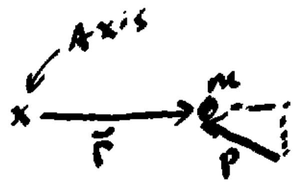
$$\sim mr^2\omega \sim r\,mr\omega \sim rmv \sim rp$$

vs.

Angular Momentum is a vector $\equiv \vec{L}$

$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{V} = I\vec{\omega}$ — same direction as $\vec{\omega}$





Again cross product projects out the component of $\vec{P}$ $\perp$ to $\vec{r}$

$$\vec{L} = \vec{r} \times \vec{p}$$

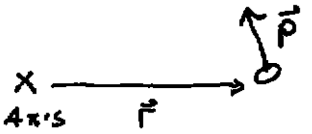
$$\frac{d\vec{L}}{dt} = \underbrace{\cancel{\frac{d\vec{r}}{dt} \times \vec{p}}}_{0} + \vec{r} \times \underbrace{\frac{d\vec{p}}{dt}}_{\vec{F}}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

Rate of change of Angular momentum =
torque of net force acting on object

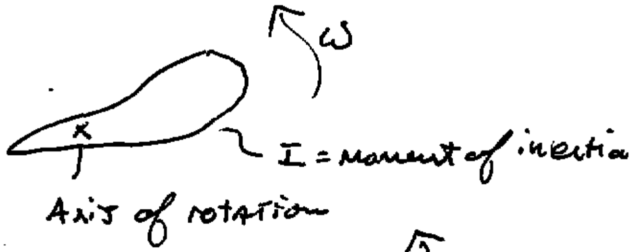
Nov 2

Angular Momentum



$\vec{L} = \vec{r} \times \vec{P}$

discrete body $\Uparrow$



$\vec{L} = I\vec{\omega}$

extended rigid body $\Uparrow$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

(just like $\frac{d\vec{P}}{dt} = \vec{F}$ in linear case)

In the absence of an $\vec{\tau}_{NET}$ $\quad \frac{d\vec{L}}{dt} = 0$

Also, Angular Momentum is Conserved just as in Linear Momentum.

$$\Sigma \vec{L}_{init} = \Sigma \vec{L}_{Final}$$

P113
S. Manly

NAME Soln key Sln

University of Rochester
Fall 1999

Problem 10 (10 pts) – zero/half/full credit:

The matter in stars is in equilibrium between the gravitational force pulling in radially and the "radiation pressure" of energy released by thermonuclear reactions pushing out. When they run out of nuclear fuel in the center (core), the radiation pressure is reduced and they collapse.

Under gravitational collapse, the radius of a spinning spherical star of uniform density shrinks by a factor of 2, with the resulting increased density remaining uniform throughout as the star shrinks. What will be the ratio of the final angular speed ω_2 to the initial angular speed ω_1? Select the correct answer below. You must show your supporting work to get credit.

$\omega 2/\omega 1$ = (a) 2 (b) 0.5 (c) 4 (d) 0.25 (e) 1.0

Angular Momentum is Conserved

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

I for Solid sphere $= \frac{2}{5} MR^2$

$$\frac{2}{5} M R_i^2 \omega_i = \frac{2}{5} M R_f^2 \omega_f$$

$$R_i^2 \omega_i = \left(\frac{1}{2} R_i\right)^2 \omega_f$$

$$\frac{\omega_f}{\omega_i} = 4$$

JUST as in linear Momentum

Angular Momentum is Conserved!

$$\sum \vec{L}_{initial} = \sum \vec{L}_{Final}$$

Let's do a couple of example problems to get the hang of some of this New Stuff —

Example

~~An ice skater with out stretched hands spins with Angular velocity $\omega = 0.60$ rev/s~~

A student stands on a turntable with dumbbells held in outstretched hands.

Initially the student is given an angular velocity of 0.5 revolutions/second.

The student then pulls the weights in close to his/her body. What is the students Angular velocity in the new configuration?

Given info M of each dumbbell = 5 kg

Length of arm (to center of chest) ~ 0.6 m

$I_{body} \sim 0.40\ kg \cdot m^2$

Masses held close ... dist from body center = 0.2 m

use Momentum Conservation:

$L_{initial} = L_{final} \rightsquigarrow \sum I\omega$ initial $= \sum I\omega$ final

$$(I_{body} + I_{mass1} + I_{mass2})\,\omega_{initial} = (I_{body} + I_{mass1} + I_{mass2})\,\omega_{final}$$

$$\left[0.4\ kg\,m^2 + (5\ kg)(0.6\ m)^2\, 2\right]\ 0.5\ rev/s\ \ 2\pi\ \frac{rad}{rev}$$

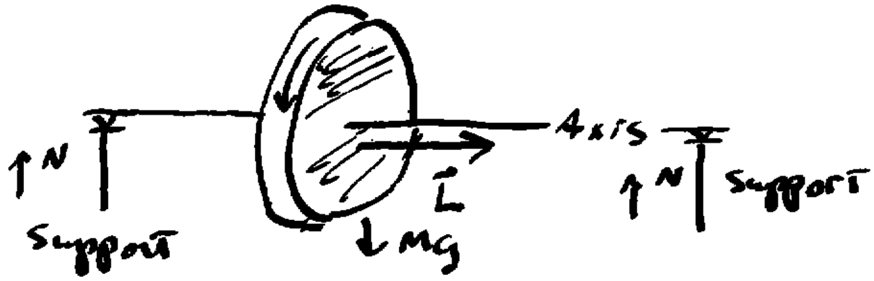
$$= \left[0.4\ kg\,m^2 + (5\ kg)(0.2\ m)^2\, 2\right]\ \omega_{final}$$

$12.56\ rad/s\ kg\,m^2 = 0.8\ kg\,m^2\ \omega_{final} \qquad \omega_{final} = 15.7\ rad/s = 2.4\ rev/s$

Precession and Gyroscopes

recall $\frac{d\vec{L}}{dt} = \vec{\tau}$

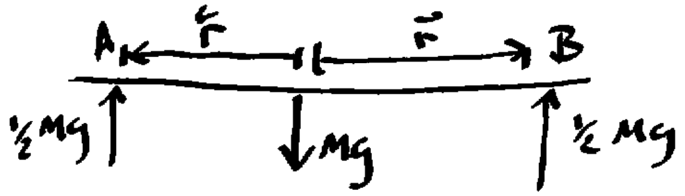
heavy disk spinning on an axis



heavy wheel spinning on horizontal axis

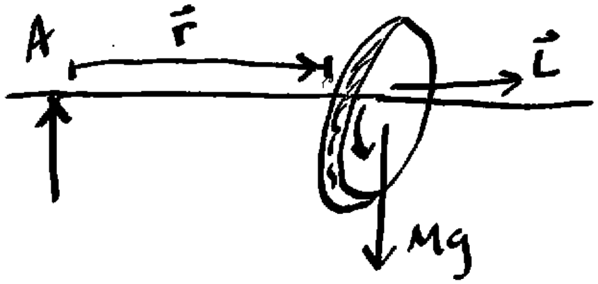
~~TAKE AWAY ONE SUPPORT.~~

System is stable



+ wheel spins

Now take away one support (remove support B)



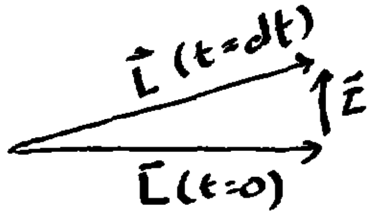
Now there is a Torque about point A

$$\vec{\tau}_A = \vec{r}\times\vec{F} = \vec{r}\times M\vec{g} = rMg \text{ into page}$$

Now recall

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

view from above



$\Rightarrow$ Axis + spinning disk supported at one end and free to rotate about support does NOT fall $\Rightarrow$ IT rotates in the horizontal Plane!!

Is this weirdness or what!?

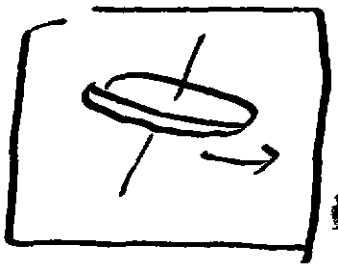
This is known as precession

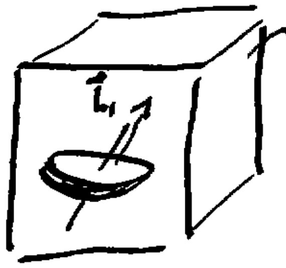
Have you ever noticed a Top spinning?

It Precesses

Imagine a system w/ a disk free to spin ~~in 3d~~ along an axis in 3d

i.e. Axis is free to rotate



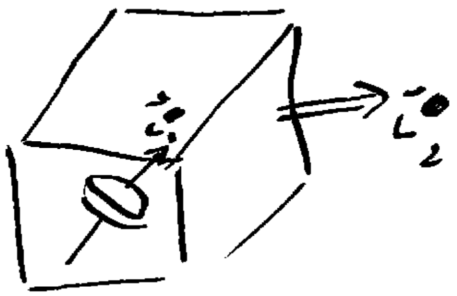


$\vec{L}_2 = 0$

Suppose <u>box</u> is rotated

what happens?

Total $\vec{L}$ is conserved



If $\vec{L}_2 \neq 0$

$\vec{L}_1$ must ~~chang~~ change to cancel out $\vec{L}_2$ in vector sum!

There is a change in the orientation of the spinning disk in response to the rotation of the box (about <u>Any</u> Axis)

Gyroscopes are a <u>priceless</u> tool for Navigation