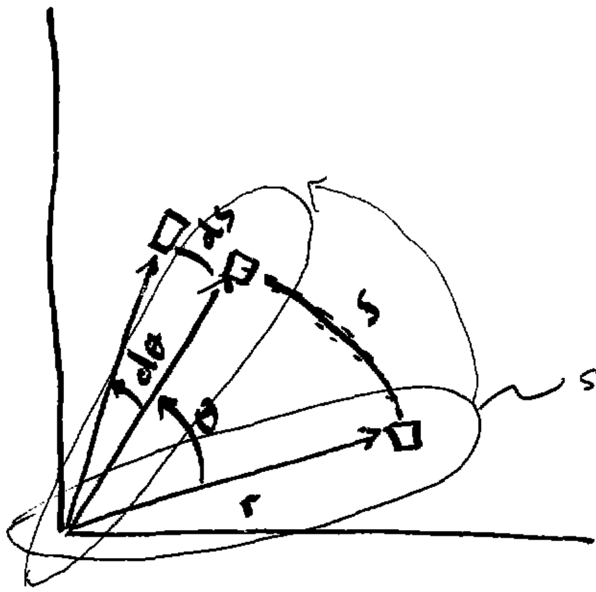
Rotational Motion

P121 Lecture
3/13/01



Some rigid body rotates thru Angle θ

$$s = r\theta$$

Arclength = (Radius)(θ in radians)

$$ds = r\,d\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\underset{\substack{\text{Tangential}\\\text{Velocity}\\(m/s)}}{v} = r\underset{\substack{\text{Angular}\\\text{Velocity}}}{\omega} \quad \left(\text{radians}/s\right)$$

All you need remember

$s = r\theta$

$\left.\begin{array}{l} v = r\omega \\ a_t = r\alpha \end{array}\right\}$ follow

$$\underbrace{\frac{d^2s}{dt^2}}_{\substack{\text{Tangential}\\\text{Acceleration}}} = \frac{dv}{dt} = r\underbrace{\frac{d^2\theta}{dt^2} = r\frac{d\omega}{dt}}_{\substack{\text{Angular}\\\text{Acceleration}\\\alpha}}$$

$$\frac{d\theta}{dt} = \omega \qquad\qquad \frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha\, dt$$

$$d\theta = \omega\, dt$$

$$\int_{\theta_0}^{\theta} d\theta = \int_{t_0}^{t} \omega\, dt \qquad\qquad \int_{\omega_0}^{\omega} d\omega = \int_{t_0}^{t} \alpha\, dt$$

$$\boxed{\theta - \theta_0 = \int_{t_0}^{t} \omega\, dt} \qquad\qquad \boxed{\omega - \omega_0 = \int_{t_0}^{t} \alpha\, dt}$$

if α is constant

$$\omega - \omega_0 = \alpha(t - t_0)$$

$$t_0 = 0 \Longrightarrow \omega = \omega_0 + \alpha t$$

Does all this look familiar ??

$$x, v, a, t \Longleftrightarrow \theta, \omega, \alpha, t$$

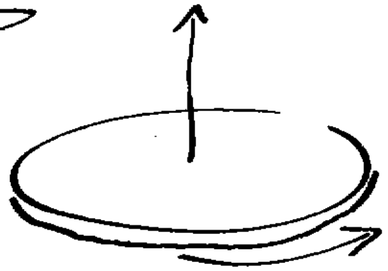
The differential eqns that relate the variables are the same. $\therefore$ The eqns that relate them will be the same.

For a = CONSTANT		For α = CONSTANT
Linear variables		Angular variables
$v = v_0 + at$	$\Longrightarrow$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2} a t^2$		$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$		$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v + v_0)t$		$\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t$

All you know about solving 1-d linear const. Acceleration problems $\longrightarrow$ carries over to

CONSTANT Angular Acceleration Problems

Example



A disk initially Rotating at 120 Rad/s slows down w/ Constant Angular Acceleration of 4.0 rad/s^2

$\Rightarrow$ How much time passes before the disk stops rotating?

$$\omega = \omega_0 + \alpha t$$

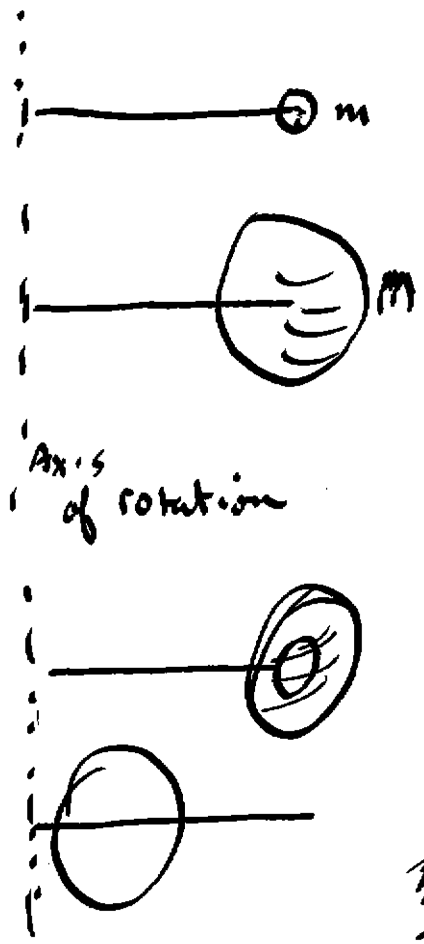
$$0 = 120 - 4.0\, t$$

$$t = 30 \text{ seconds}$$

So far relating linear to angular variables works well

Linear $\quad F = Ma$

Angular? $\quad ? = ?\, \alpha$



Which is harder to rotate?

requires more force

Which of these?

~~Also door example~~

$F = Ma$ linear

$F = ?\,\alpha$ Angular

obviously depends on M ... magnitude of body

And **Mass** **distribution**

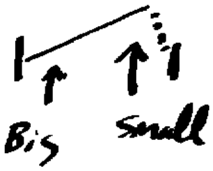
Do demos

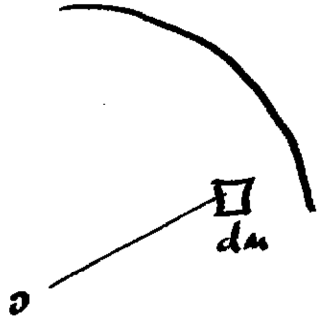
3 from Thang

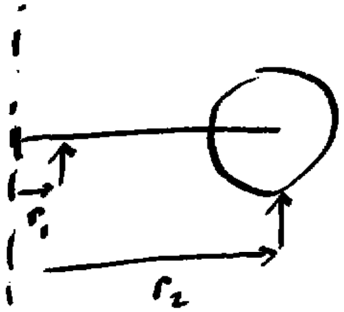
Door demo

$F = f(m, r)\,\alpha$

Do door Demo
2 students







consider Application of force
does it matter where you
Apply it

So ... α depends on Mass, dist of Mass, Force
<u>And</u> point of Application of force!

$F = ma$

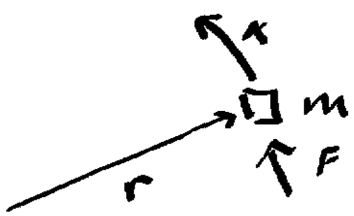
$s = r\theta$
$v = r\omega$
$a = r\alpha$

ignore Vector character for Now

$F = mr\alpha$

$rF = rmr\alpha$

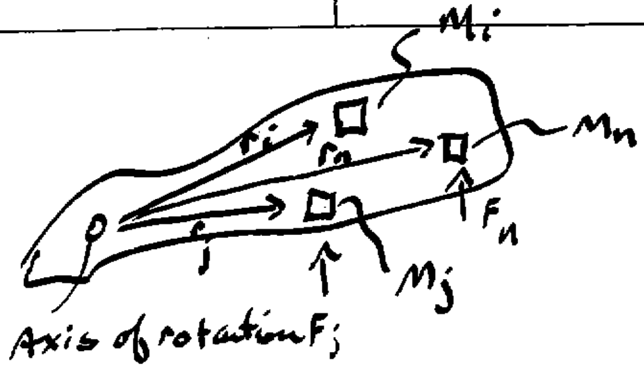
$rF = Mr^2\alpha$



Angular Acceleration

Angular Force
Known as Torque
τ

Angular Mass
Known as <u>Moment</u> of <u>Inertia</u> of Mass m
I



$$\left(\sum r_i F_i\right) = \left(\sum m_i r_i^2\right)\alpha$$

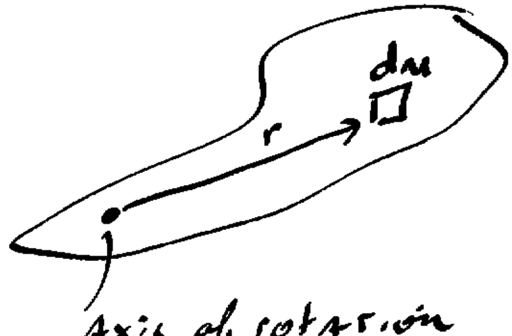
Net "angular" force

net "angular" mass

$$\sum \tau_i = \underbrace{\sum m_i r_i^2}_{I_i} \alpha$$

$$\tau_{NET} = \sum m_i r_i^2 \alpha = I\alpha \qquad \boxed{\tau_{NET} = I\alpha}$$

go to continuous limit



$\cancel{d\tau = r^2\,dm\,\alpha}$

$$dI = r^2 dm$$ dI for differential element

$$I = \int_V r^2 dm$$ Total I from integral over volume

Vector vs. Scalar?

Does I have a direction?

⇒ Scalar ... just like regular mass