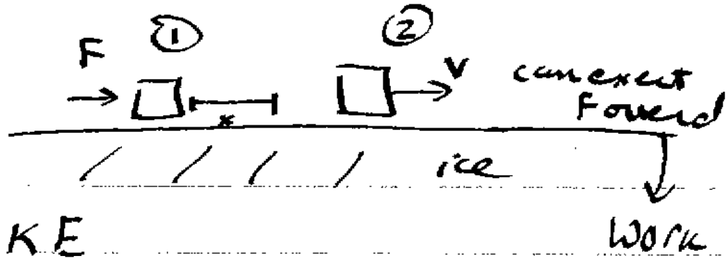
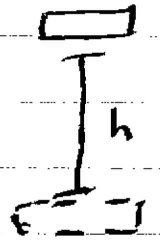


So,

$$\begin{tikzcd} \text{Work} \arrow[r] & \text{KE} \arrow[l, bend left] \end{tikzcd}$$

Also lift Book up a distance h



$F \cdot d = mgh$ amount of work

Now let it go – falls a distance h w/ $a = g$

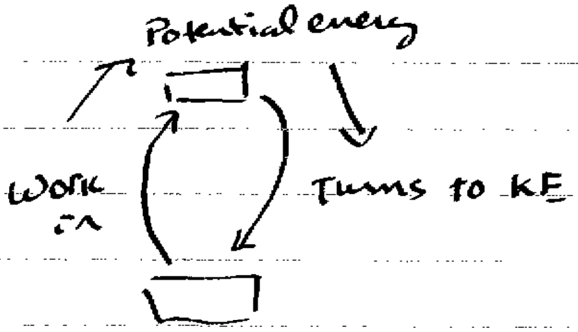
$\cancel{h = \frac{1}{2}gt^2}$ $\quad v^2 = v_0^2 + 2ah$

$v^2 = 2gh$

F/m

$\frac{1}{2}mv^2 = \cancel{\frac{1}{2}}m\,2gh$

$\frac{1}{2}mv^2 = Fh$ = work done by gravity



In general

$$\boxed{E_{TOTAL} \text{ is Conserved}}$$

what do I mean by "is conserved"?

$$\left(\sum_i E_i\right)_{initial} = \left(\sum_j E_j\right)_{Final}$$

The form of the energy may change

Sometimes is NOT so obvious – consider pushing a car

$$W_{\substack{push\\car}} = \Delta PE_{\substack{car\\height}} + \Delta KE_{\substack{car\\motion}}$$

$$+ W_{friction} + \text{heat energy}$$

$$+ \text{sound energy}$$

$$+ \cdots$$

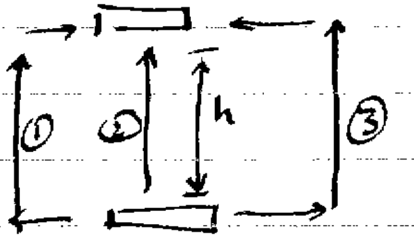
In the real world

$$W_{\substack{Push\\Car}} > \Delta PE + \Delta KE$$

Most of the time

useful to look at situations where other losses are NOT present.

Consider Slide w/ NO friction or



lift book by 3 separate paths

Work done in each case = Mgh

ΔPE in each case = Mgh

W = ΔPE + ΔKE

The change in potential energy is <u>Path Independent</u>

Due to a <u>Conservative Force</u>

With a Conservative force

Work = ΔPE + ΔKE

one can Study energy flow

Conser. Systems that are commonly studied

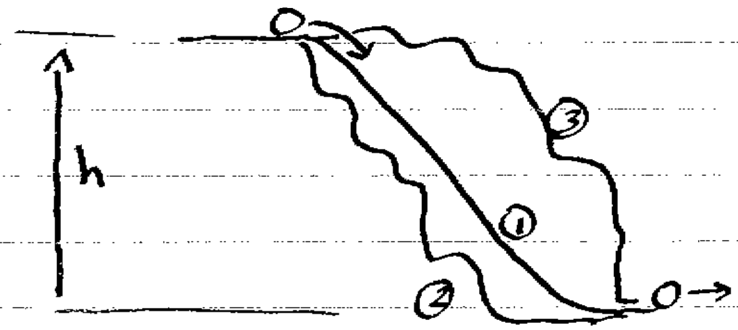
* Springs ← force is NOT CONSTANT

Gravity ← force const over small dist. at earth's surface

Very Impt. (chemical bonds + other things)

$\frac{1}{r^2}$ force in general

Consider a slide with friction



Will v at bottom be the same for all paths?

i.e. will KE at Bottom be the same?

All start w/ same PE

paths are different $F_{friction}$ varies and acts over different distances

$\Rightarrow$ KE at bottom varies depending on path

$\rightsquigarrow$ Non conservative situation

Is energy conserved?

IS Work = ΔPE + ΔKE

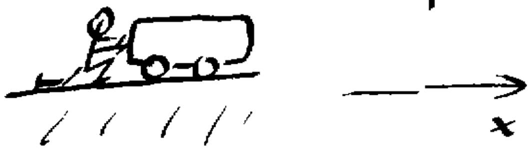
$0 = \Delta PE + \Delta KE$

$mgh = \frac{1}{2}mv^2$?

No because have $\Delta W_{friction}$ in eqn too!

Suppose you push cart ... but now you do NOT push w/ a constant force

$\rightarrow F(x)$



F depends on x

Suppose $F_{push} = (9x^2 - 2x)$ N

Start at $x = 0$, Push to $x = 10$ m $(D = 10)$

$$W = \int_0^{10} F \cdot dx = \int_0^{10} (9x^2 - 2x)\, dx$$

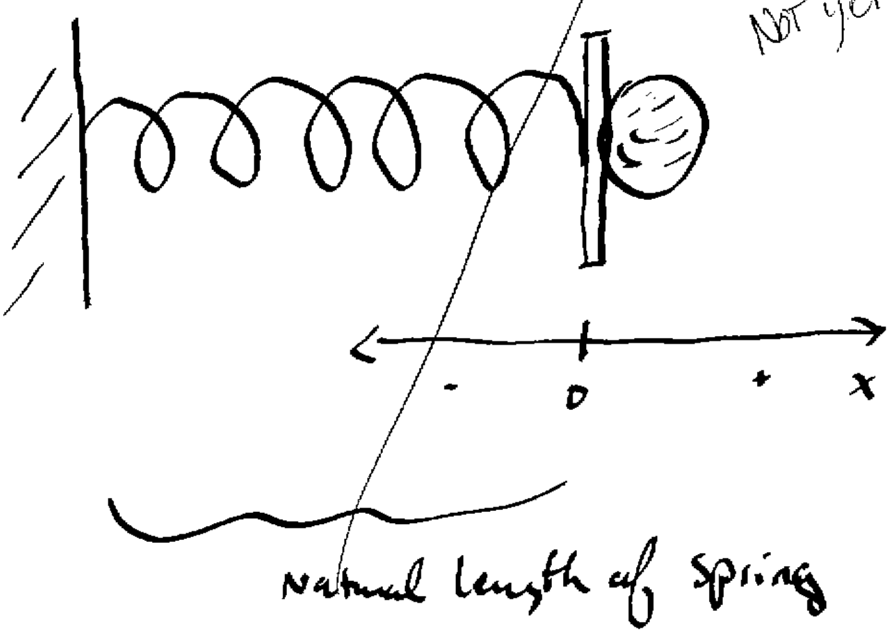
$$W = \frac{9x^3}{3}\Big|_0^{10} - x^2\Big|_0^{10} = 3000 - 100 = 2900 \ \overbrace{Nm}^{\text{Joules}}$$

NOT a realistic # but you see what I mean ---

Variable ~~foces~~ forces

The Simple Spring

NOT YET



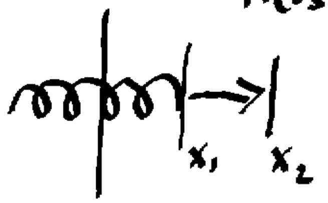
$F = -kX$

$\vec{F} = -k(\vec{x} - \vec{x}_0)$

Natural position of spring end

Step back look at Springs

Signs of work + Potential Energy again
This Is very subtle



Let $x_0 = 0$ for simplicity

$x_0 \equiv$ natural length

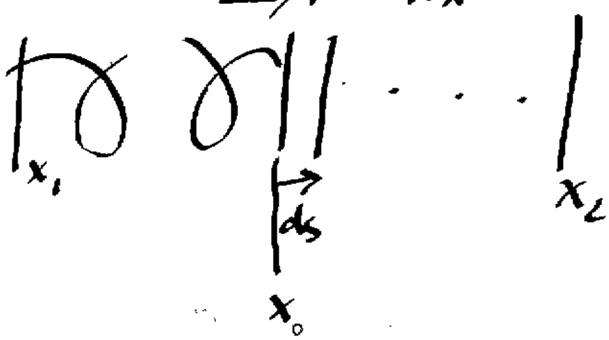
$\vec{F} = -k(x - x_0) = -k\vec{x}$ — sign gives direction "restoring force"

on known as Hooke's Law

What is work done ~~by~~ on spring to elongate from x_1 to x_2

Consider arbitrary point between x_1 and x_2

$\rightarrow F = kx$



Work done on spring ∴ consider force that is pulling on spring.

F and ds are in the same direction

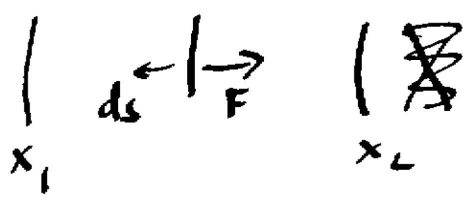
$\therefore \ \vec{F}\cdot d\vec{s} = F ds = F dx = kx dx$ — no need to put sign because you've done that taking dot product already

$$W = \int_{x_1}^{x_2} \vec{F}\cdot d\vec{s} = \int_{x_1}^{x_2} kx\,dx = k\frac{x^2}{2}\Big|_{x_1}^{x_2} = \frac{k}{2}x_2^2 - \frac{k}{2}x_1^2 \quad (+)$$

$x_2 > x_1$ ∴ work done on spring is (+)

Work done on spring going from x_2 to x_1 (Slowly)

all the same ... but



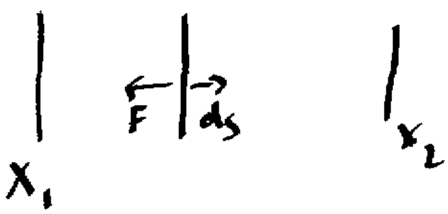
$\therefore \ \vec{F}\cdot d\vec{s} = -kx dx$

$\therefore$

$$W = \frac{k}{2}x_1^2 - \frac{k}{2}x_2^2 \quad (-)$$

Work done on spring = − Work done by Spring

~~that~~ consider work done by spring from x_1 to x_2



$\vec{F} \cdot d\vec{s} = -kx\,dx$

$\Rightarrow W = \frac{k}{2} x_1^2 - \frac{k}{2} x_2^2$ $\quad -$

define elastic Potential Energy of a spring

$U \equiv \frac{1}{2} k x^2$ Ability to do WORK!

I do work on Spring $\quad x_1 \longrightarrow x_2 \quad$ Stretching it

Δ Potential Energy $= \frac{k x_2^2}{2} - \frac{k x_1^2}{2}$ is $+$

= Work I did on spring

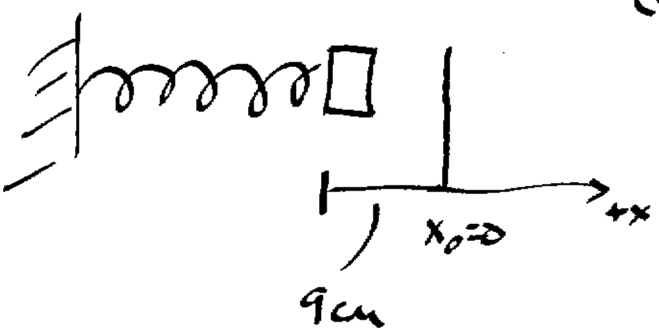
I do work on spring letting it slowly relax from $x_1 \leftarrow x_2$

Δ Pot energy $= \frac{k x_1^2}{2} - \frac{k x_2^2}{2}$ is $-$

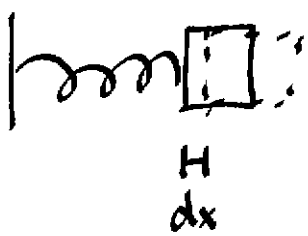
Also = to work I did in this case

A person places a 2 kg block against a spring (w/ spring constant $k = 200$ N/m) and compresses the spring 9 cm from its Natural length --- No Friction in problem.

(a) Find the work done by the ~~cotton~~ person and the work done by the spring

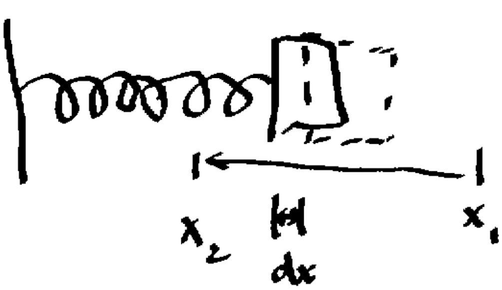


$$\vec{F}_{spring} = -k(\vec{x}-\vec{x}_0) \Rightarrow |F_{spring}| = kx \text{ To Right}$$



$$dW = F_{person}\, dx$$

$$\int dw = W = \int_{x_1}^{x_2} F\, dx$$



$$W = \int_{x_1}^{x_2} kx\, dx$$

$$= k\frac{x^2}{2}\Big|_{x_1}^{x_2} = \frac{1}{2}k\underbrace{x_2}_{.09\text{ m}}{}^2 - \frac{1}{2}k\underbrace{x_1}_{0}{}^2$$

$$= \cancel{\tfrac{1}{2}kx_2^2}\ 1.22 \text{ J}$$

work done by ~~[illegible]~~ person on spring $\longrightarrow$

Work done by spring on ~~[illegible]~~ person will be -1.22 J

(b) Block is released and leaves spring when it is at its natural length

Find the speed of the block in this case

Energy stored in spring is released (put there by person's work)

$\Rightarrow$ Turned to kinetic Energy

$$\frac{1}{2}k(x_2-x_1)^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k}{m}}\, x$$