Linear Momentum - Momentum Conservation

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

define $m\vec{v} \equiv \vec{P} \equiv$ momentum

vector quantity

$$P_x = mv_x$$

$$P_y = mv_y$$

$$P_z = mv_z$$

Typically $\frac{d(m\vec{v})}{dt} \Longrightarrow$ m constant

$$\frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt}$$

NOT always !

Consider a Rocket ... Burns fuel

$$\frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

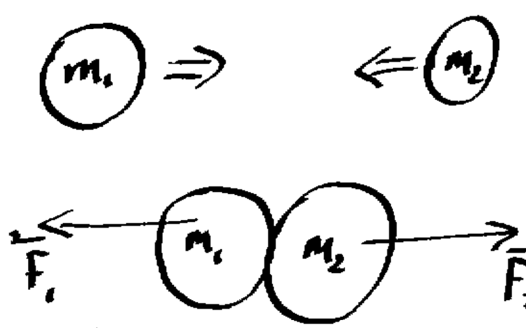
$$\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow d\vec{P} = \vec{F}dt \Rightarrow \int_{P_1}^{P_2} d\vec{P} = \int_{t_1}^{t_2} \vec{F}dt$$

$$\Delta\vec{P} = \vec{P}_2 - \vec{P}_1 = \int_{t_1}^{t_2} \vec{F}dt$$

1 ≡ initial
2 ≡ Final
here

$\equiv J \equiv$ impulse

Consider two bodies colliding



not complicated

complicated

$$\Delta\vec{P}_1 = \int_{t_i}^{t_f} \vec{F}_1\, dt \qquad \Delta\vec{P}_2 = \int_{t_i}^{t_f} \vec{F}_2\, dt$$

but we know from Newton's 3^{rd} Law that $\vec{F}_1 = -\vec{F}_2$ at all times

$$\therefore \underbrace{\int_{t_i}^{t_f} \vec{F}_1\, dt}_{\Delta\vec{P}_1} = \int_{t_i}^{t_f} (-\vec{F}_2)\, dt = -\underbrace{\int_{t_i}^{t_f} \vec{F}_2\, dt}_{\Delta\vec{P}_2}$$

$$\therefore \Delta\vec{P}_1 = -\Delta\vec{P}_2 \quad \text{or} \quad \Delta\vec{P}_1 + \Delta\vec{P}_2 = 0$$

Momentum of individual parts has changed

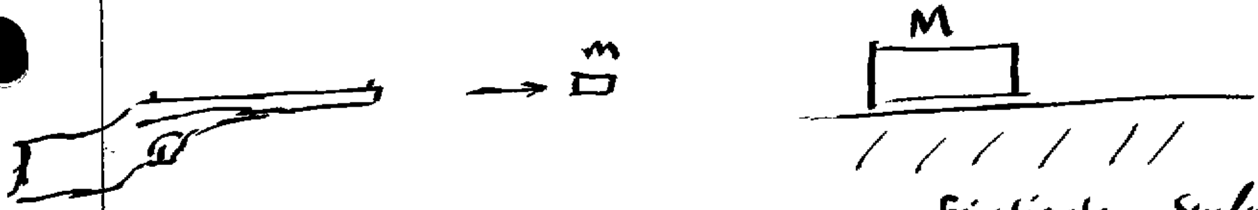
Σ changes in the momentum $\rightarrow 0$!

Momentum Conservation

If there are no external forces acting on a system, the total momentum of that system is conserved.

-or- $\Sigma \Delta\vec{P} = 0$ -or- $\Sigma \vec{P}_i = \Sigma \vec{P}_F$

Example



Frictionless Surface

bullet fired w/

unknown velocity $\vec{V}_{bullet}$ ← Find This

embeds in block "instantaneously"

block + bullet move, $\vec{V}_f$ Measured (Block + Bullet combo)

momentum cons ⇒

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$m_{bullet} \vec{V}^{\,i}_{bullet} + M_{block} \underbrace{V^{\,i}_{block}}_{0} = (m+M)\vec{V}^{\,f}_{block/bullet}$$

$$\vec{V}^{\,i}_{bullet} = \left(\frac{m+M}{m}\right)\vec{V}^{\,f}_{block/bullet}$$

Is energy Conserved? → yes

$KE_{bullet} = KE_{bullet + Block}$? ⇒ No

(No change in Pot energy here)

Frictional energy loss to Stop bullet

⇒ Known as an inelastic Collision

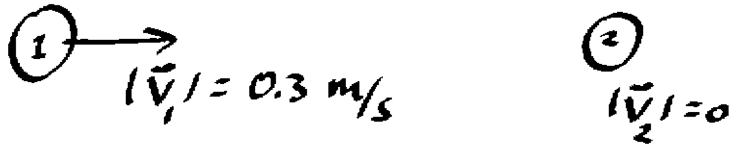
in other words

$$\frac{1}{2} m v_i^2 \neq \frac{1}{2}(m+M) v_f^2$$

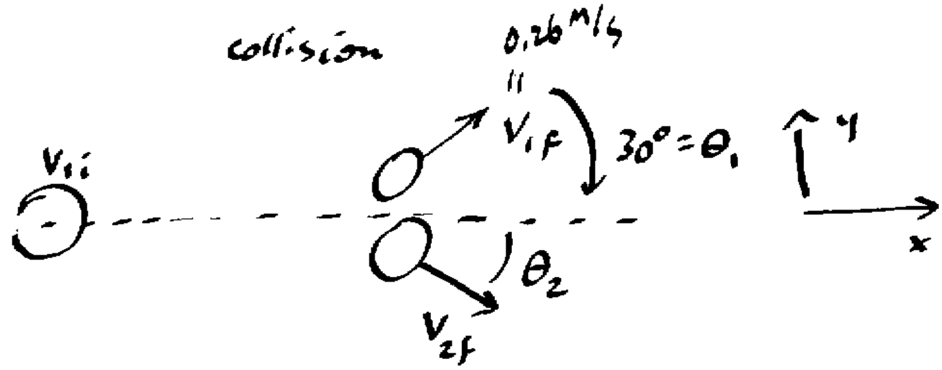
Example

Consider The collision of two hockey Pucks each of Mass 0.1 kg

initially



Final



$V_{1f} = 0.26\ m/s$

What is the final ~~speed~~ velocity of the 2^{ND} puck ?

$\Sigma \vec{P}_i = \Sigma \vec{P}_f$ ← vector eqn

Same as $\left(\begin{array}{c} \Sigma P_{ix} = \Sigma P_{xf} \\ \text{-and-} \\ \Sigma P_{yi} = \Sigma P_{yf} \end{array} \right)$ Momentum Conserved for each component

x eqn

$$m_1 V_{1ix} + m_2 \underbrace{\cancel{V_{2ix}}}_{0} = m_1 \underbrace{V_{1fx}}_{V_{1f}\cos 30} + m_2 \underbrace{V_{2fx}}_{V_{2f}\cos\theta_2}$$

$$\text{(I)} \quad m_1 V_{1ix} = m_1 V_{1f} \cos 30 + m_2 V_{2f} \cos\theta_2$$

y eqn

$$m_1 \underbrace{\cancel{V_{1i_y}}}_{0} + m_2 \underbrace{\cancel{V_{2i_y}}}_{0} = m_1 \underbrace{V_{1f_y}}_{V_{1f}\sin 30} + m_2 \underbrace{V_{2f_y}}_{-V_{2f}\sin\theta_2}$$

(II) $\quad V_{2f} \sin\theta_2 = V_{1f} \sin 30$

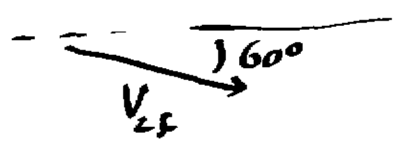
I $\quad (.1)(0.3) = (.1)(0.26)\cos 30 + (0.1)\, V_{2f} \cos\theta_2$

II $\quad V_{2f} \sin\theta_2 = (0.26) \sin 30$

2 eqns, 2 unknowns V_{2f}, θ_2

Should find $\quad |\vec{V}_{2f}| = 0.15 \text{ m/s}$

$\theta_2 = 60°$ down from horizontal



Is the collision "elastic"?

initial KE $= \frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2}(0.1)(0.3)^2 = 4.5 \times 10^{-3}$ J

final KE $= \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

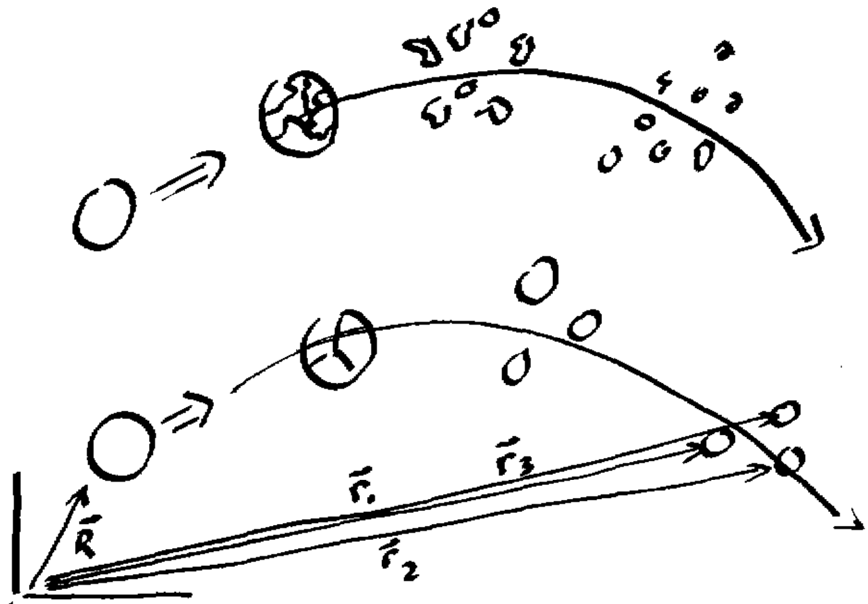
$= \frac{1}{2}(0.1)(0.26)^2 + \frac{1}{2}(0.1)(0.15)^2 = 4.5 \times 10^{-3}$ J

Yes collision is elastic.

Kinetic Energy is conserved!

~~October 19~~

Center-of-Mass



An object moving along at position $R(t)$ breaks into 3 (or more) objects w/ positions described by vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$

Momentum Conservation

$$\Longrightarrow \quad \Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt}$$

$$\frac{d}{dt}(M\vec{R}) = \frac{d}{dt}(m_1\vec{r}_1) + \frac{d}{dt}(m_2\vec{r}_2) + \frac{d}{dt}(m_3\vec{r}_3)$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3$$

break into component

$$\begin{cases} MX = m_1x_1 + m_2x_2 + m_3x_3 \\ MY = m_1y_1 + m_2y_2 + m_3y_3 \\ MZ = m_1z_1 + m_2z_2 + m_3z_3 \end{cases}$$

-or-

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$$

Similar for Y, Z

These are Known as Center-of-Mass coordinates

Can consider the system of 3 bodies equivalent to One body of Mass $M = m_1 + m_2 + m_3$ ~~at a position~~ Concentrated at the center-of-mass position

Center-of-Mass coordinates:

Mass weighted Average position of a system of particles

Consider N masses

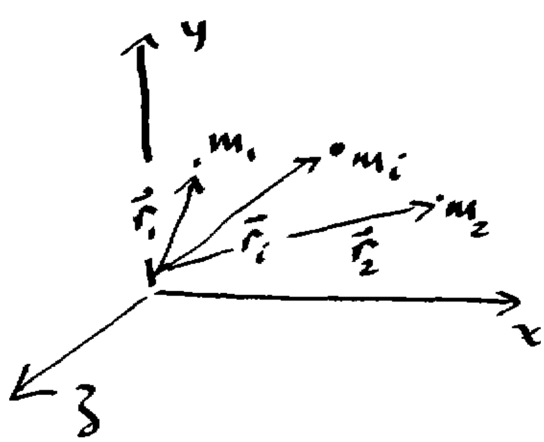
$$X_{c.m.} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$$

$$Y_{c.m.} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$$

$$Z_{c.m.} = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$$

-or-

$$\vec{R}_{c.m.} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$



Easy to use for a system of discrete particles