We've discussed work + different forms of energy
Often the time distribution of the input/output of work/energy is important

Ave. Power $\equiv \dfrac{\Delta W}{\Delta t}$

instantaneous Power $= \dfrac{dW}{dt}$

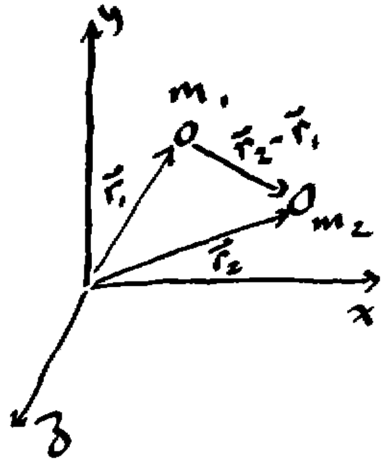
unit of Power is the Watt $\equiv$ J/s

Ex: A 100 Watt light bulb is one that converts 100 Joules of electrical energy into heat and light energy every second.

A closer look at Gravity and Grav. Pot. energy

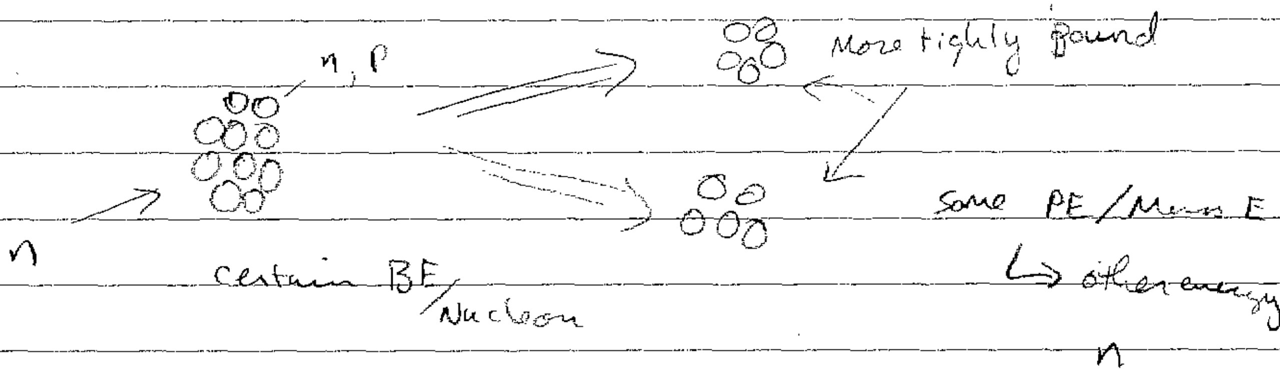
$$F_g = \frac{G m_1 m_2}{r^2}$$ Attractive along line joining "center of masses"

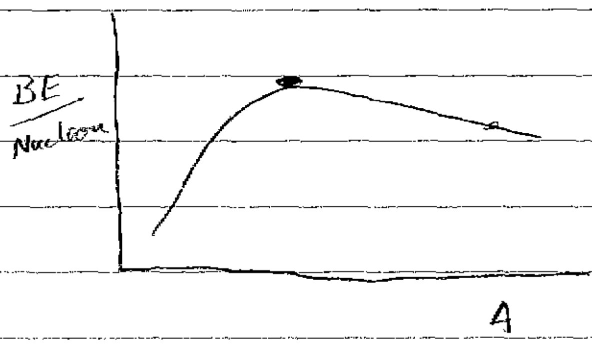
More formally written



$$\vec{F}_{1\,on\,2} = -\frac{G m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^2} \underbrace{\frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}}_{\hat{r} \text{ from 1 to 2}}$$

Power - Nuclear Fission





3000 MW Power plant

200 MeV released per fission

$\frac{1.6 \times 10^{-19}\ J}{eV}$

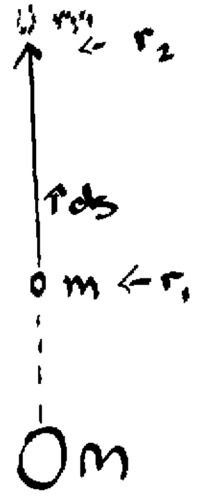
$200 \times 10^{6}\ eV \times 1.6 \times 10^{-19}\ \frac{J}{eV} = 3.2 \times 10^{-11}\ J/\text{fission}$

$3000 \times 10^{6}\ J/s \times \frac{1}{3.2 \times 10^{-11}}\ \frac{\text{fissions}}{\text{Joule}} = 9.4 \times 10^{19}\ \text{fissions}/s$

$\frac{9.4 \times 10^{19}\ \text{Nuclei}}{s} \times \frac{1}{6.02 \times 10^{23}}\ \frac{\text{mole}}{\text{nuclei}} \times 235\ \frac{g\ \text{Uranium}}{\text{Mole}} = 0.04\ g\,U/s$

$1\ yr = 60 \times 60 \times 24 \times 365 = 3 \times 10^{7}\ s \Rightarrow 1000\ kg\ U/\text{year}$

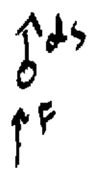
Now ... do some for gravity as we did for spring



move mass m slowly away from mass M along radial direction

What is the work done by me to ~~gravity on~~ move M m?

$$W = \int \vec{F}\cdot d\vec{s} = \int_{r_1}^{r_2} \vec{F}\cdot d\vec{r}$$



$$|d\vec{s}| = |d\vec{r}|$$

$$W = \int_{r_1}^{r_2} \cancel{-}\,\frac{GMm}{r^2}\,dr$$

dot product gives "−" due to $\vec{F}_{grav}$ and $d\vec{s}$ pointing in opposite directions

$$W = \cancel{-}\,GMm \int_{r_1}^{r_2} \frac{1}{r^2}\,dr$$

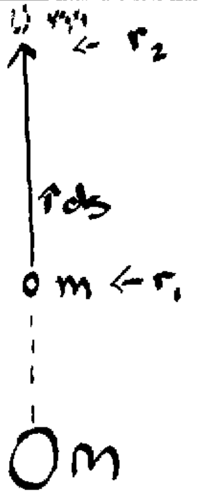
$$W = -\frac{GMm}{r}\Big|_{r_1}^{r_2} = -\frac{GMm}{r_2} + \frac{GMm}{r_1}$$ (+) #1

Recall

$$F = \frac{G m_1 m_2}{r^2} \Rightarrow \text{on earth } F = \boxed{\frac{G M_E}{R_E^2}}\, m$$

$$g = 9.8\ m/s^2$$

Now... do same for gravity as we did for spring



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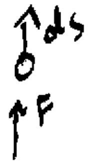

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defining grav Potential energy of system

$$PE = -\frac{GMm}{r}$$

when I move particle out from

r_1 to r_2

$$\Delta PE = -\frac{GMm}{r_2} + \frac{GMm}{r_1} \quad \oplus$$

sign chosen to make this $\oplus$

Particle ~~but~~ near a Mass

Particle has 0 potential Energy at ∞

" − potential energy at finite r

but ΔPE going from small r to larger r is $\oplus$

$$\Delta PE = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$

$$\Delta PE = \frac{GMm(r_2 - r_1)}{r_1 r_2}$$

let r_1 = surface of Earth (radius) $= R_E$ $\qquad M = M_E$

r_2 = small distance above surface of Earth

$\therefore r_2 = r_1 + \Delta r$ where $\Delta r \ll r_1$

$$\Delta PE = \frac{GM_E m\,(R_E + \Delta r - R_E)}{R_E(R_E + \Delta r)} \simeq \underbrace{\frac{GM_E}{R_E^2}}_{\equiv g} m\,(\underbrace{\Delta r}_{h}) = mgh \;!$$

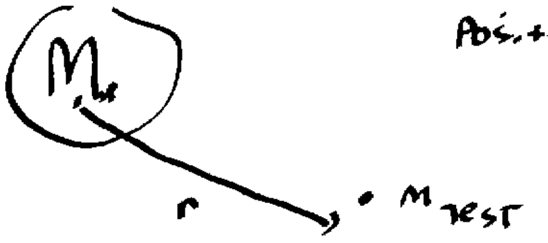
Our concept of gravity comes about from thinking about the Force of one object on another

what is the condition in space created by M_1 that causes a force on M_2 ??

Useful to think of the space around M_1 independent of any other mass. We say M_1 creates a condition in the space such that if a "TEST Mass" were there it would feel a gravitational force due to M_1.

We say M_1 ~~has~~ sets up a "gravitational Field" in the space surrounding it ... out to $r = \infty$.

Suppose we place a little "test mass" of small ~~arbitrary~~ mass in some position



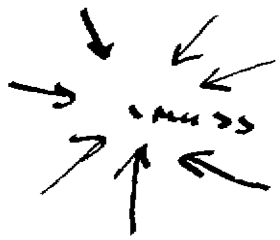
$$\vec{F}_{\text{of } M \text{ on } M_{TEST}} = \frac{G M M_{TEST}}{r^2} \hat{r}$$

$$\frac{\vec{F}}{M_{TEST}} = \frac{\overrightarrow{\text{Force of } M \text{ on a mass}}}{\text{unit mass}} = -\frac{GM}{r^2} \hat{r} \equiv \text{Gravitational Field}$$

it is a vector field

$$\frac{\vec{\text{Force}}}{\text{unit TEST MASS}} = \vec{g} \equiv \text{gravitational field}$$

vector field at each point



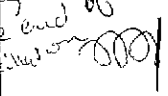
can visualize w/ "lines of force" that represent how a test mass would move at each point

$$\vec{F} = m\vec{g}$$

Enough of Gravity for Now:

Think about Energy Conservation in Conservative Systems

spring at one end of oscillation

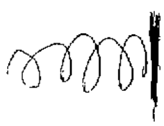


PE MAX $\frac{1}{2}mv^2$ $v=0$

KE Min $\frac{1}{2}kx^2$

If No additional work in or out

middle

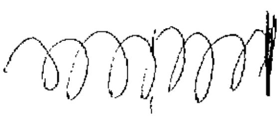


KE MAX

PE Min

$0 = \Delta PE + \Delta KE$

$\Delta PE = -\Delta KE$



other end of oscillation

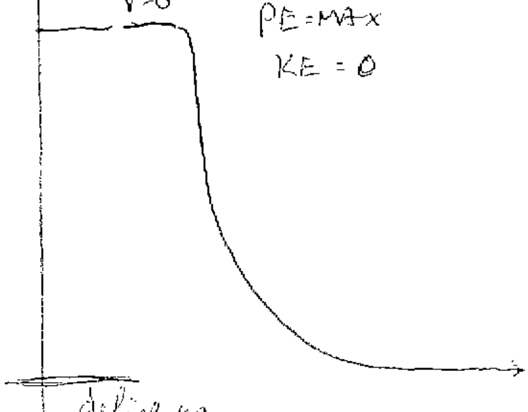
PE MAX

KE Min

$V=0$

PE = MAX

KE = 0



$V = \text{MAX}$

KE MAX

PE = 0

define no zero of PE

Consider system w/ | Movement along x

$$F_x \Delta x = -\Delta PE \equiv -\Delta U$$

$$F_x = -\frac{\Delta U}{\Delta x} \xrightarrow[\text{in limit of small } \Delta s]{} F_x = -\frac{dU}{dx}$$

Generalize to 3d

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\vec{F} = -\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}$$

but F is fn of x, y, z $\therefore$ can't use normal derivatives

Must use partial derivatives

Let $B(x,y,z)$ $\quad \dfrac{\partial B}{\partial x} = \left.\dfrac{dB}{dx}\right|_{y,z}$ Treated as constants

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

Why do you care?

⇒ In the end physics often boils down to determining $\vec{F}$ at a point or in a region

⇒ If we know $\vec{F}$, we know motions of particles

$\vec{F}$ is a vector "field" — Sometimes hard to determine for gravity $\quad \frac{\vec{F}}{m} = \vec{g} \equiv$ gravitational field

For Electromagnetism

$$\frac{\vec{F}}{q} = \vec{E} \equiv \text{Electric field}$$

Knowing $\vec{F}$ is to know $\vec{g}$ or $\vec{E}$

$\Rightarrow$ These are <u>vector</u> fields : often hard to calculate

<u>But</u> we just learned if we can understands the Potential Energy function in space we can derive the Force (or $\vec{g}$ or $\vec{E}$)

Potential Energy function is a <u>Scalar</u> field often easier to understand + get than $\vec{E}$ or $\vec{g}$ directly!

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

define $\quad \vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

vector operator "del" $\equiv$ gradient

$\vec{F} = -\vec{\nabla} U$ ... We say the Force is the negative gradient of the Potential Energy function!