P113 University of Rochester NAME Soln Key - SAM
S. Manly Fall 2001

Final Exam (December 18, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (10 pts):

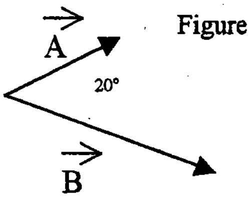


Figure 1

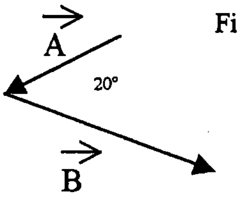


Figure 2

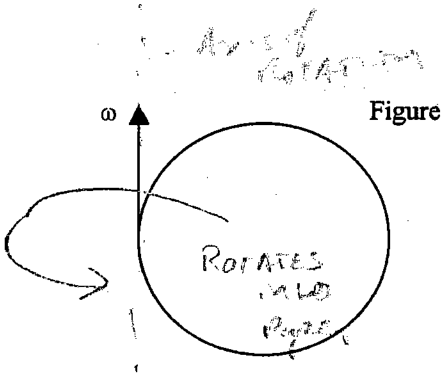


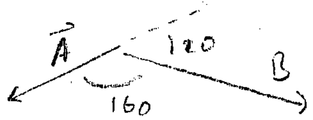
Figure 3

Assume the magnitude of vector A in the figures above is 8 and the magnitude of vector B in the figures above is 3.

2 pts a) Evaluate $\vec{A} \bullet \vec{B}$ in figure 1 above.

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos 20 = 3 \cdot 8 \cdot \cos 20 = 22.5$

2 pts b) Evaluate $\vec{A} \bullet \vec{B}$ in figure 2 above.



$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos(160) = -22.5$

OKAY if just write -22.5 Knowing $(-\vec{A}) \cdot \vec{B} = -\vec{A} \cdot \vec{B}$

2 pts c) Evaluate $\vec{A} \times \vec{B}$ in figure 1 above.

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin 20$ into paper

$|\vec{A} \times \vec{B}| = 8.2$

Direction is into paper

2 pts d) Evaluate $\vec{A} \times \vec{B}$ in figure 2 above.

SAME AS Above but opposite in direction

$|\vec{A} \times \vec{B}| = 8.2$

Direction is out of paper

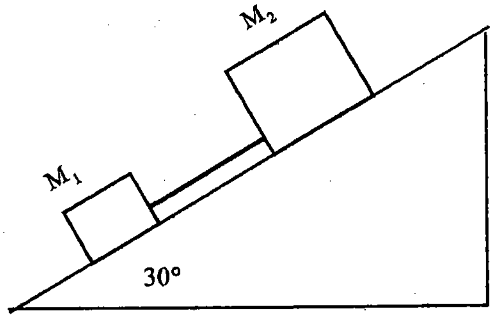
2 pts e) Figure 3 above shows a rotating object. The angular velocity vector for this system is shown. Describe and/or sketch the way the object moves (rotates) on the diagram. Label the axis of rotation.

From above object rotates counterclockwise

Problem 2 (10 pts):

Two blocks made of different materials connected together by a thin cord, slide down a plane ramp inclined at an angle θ=30° to the horizontal as shown in the sketch below. The masses of the blocks are m_1=5 kg and m_2=6 kg and the coefficients of friction are μ_1=0.2 and μ_2=0.3.

a) Determine the acceleration AND TENSION of the blocks.



⊥ eqns

$N_1 = M_1 g \cos 30$

$N_2 = M_2 g \cos 30$

$f_1 = N_1 \mu_1$

$f_2 = N_2 \mu_2$

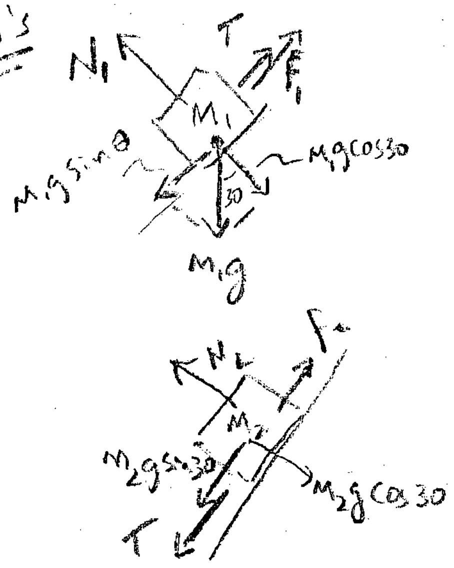
∥ eqns

$m_1 a = M_1 g \sin\theta - T - f_1$

$m_2 a = M_2 g \sin\theta + T - f_2$

rewrite, subst. in for f_1 and f_2

FBD's



~~b) Determine the tension in the cord:~~

$M_1 a = M_1 g \sin\theta - T - M_1 g \cos\theta \mu_1$

$m_2 a = M_2 g \sin\theta + T - m_2 g \cos\theta \mu_2$

$$(M_1 + M_2) a = (M_1 + M_2) g \sin\theta - (M_1\mu_1 + M_2\mu_2) g \cos\theta$$

$$a = g \sin\theta - \left[\frac{M_1\mu_1 + M_2\mu_2}{M_1 + M_2}\right] g \cos\theta = \overbrace{9.8 \sin 30}^{4.9} - \overbrace{\left[\frac{(5)(.2) + (6)(.3)}{(5+6)}\right] 9.8 \cos 30}^{2.16}$$

$$\frac{1 + 1.8}{11} = 0.25$$

$$\boxed{a = 2.74\ \mathrm{m/s^2}}$$

sub into eqn above to find T

$$T = m_1 g \sin\theta - M_1 g \cos\theta \mu_1 - m_1 a$$

$$T = \underbrace{(5)(9.8) \sin 30}_{24.5} - \underbrace{(5)\,9.8(\cos 30)\,0.2}_{8.49} - \underbrace{(5)(2.74)}_{13.7} = \boxed{2.3\ N = T}$$

Problem 3 (10 pts):

Some frat boy named Chester looks around for a game of Beruit after a long, hard week of studying physics. As usual, all of his friends are relaxing with poetry readings and games of chess. They have no interest in Beruit. So, Chester decides to play himself. He places a cup before him on a table and tosses a Ping-Pong ball straight up in the air. It lands in the cup.

Chester releases the ball with an initial velocity upward of 0.75 m/s at a point 0.3 m above the surface of the table. Assume air friction is negligible.

(a) How high does the ball rise above the table surface?

$V = 0$ at Top of Motion

$$V^2 = V_0^2 + 2a(y - y_0)$$

$$0 = (0.75)^2 - 2(9.8)(y - 0.3)$$

$$\boxed{y = 0.33 \text{ m}}$$

(b) How long is the ball in the air? (Assume the bottom of the cup is at the height of the surface of the table.)

$$y = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2$$

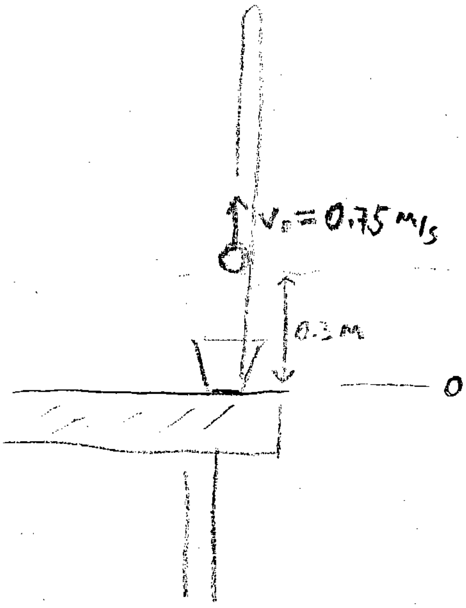
$$0 = 0.3 + (0.75)t - \frac{9.8t^2}{2}$$

$$t^2 - 0.15t - 0.06 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{0.15 \pm \overset{0.51}{\sqrt{.15^2 + (4)(.06)}}}{2} = \begin{matrix} -0.18 s \\ \text{or} \\ 0.33 s \end{matrix}$$

$$\boxed{t = 0.33 \text{ s}}$$

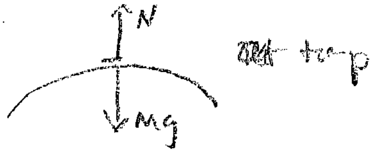


Problem 4 (10 pts):

A ferris wheel 23 m in diameter rotates once every 12 s. What is the fractional change in a person's apparent weight

a) at the top of the motion (as compared to their weight at rest)

At top $Mg - N = \frac{mv^2}{R}$ Circular Motion



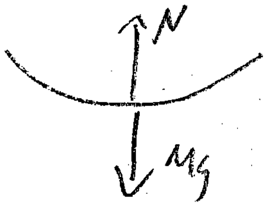
What is V? $V = \frac{2\pi R}{T} = \frac{2\pi\,23}{12} = 12.0\ m/s$

Apparent weight is N fractional change is $\frac{\overset{N}{Mg - \frac{mv^2}{R}} - \overset{\text{old N (No Motion)}}{Mg}}{Mg} = \frac{v^2}{Rg}$

Fract. change at top is $\frac{12^2}{(23)\,9.8} = -0.63$

b) at the bottom of the motion (as compared to their weight at rest)

AT BOTTOM



$N - Mg = \frac{mv^2}{R}$

$N = \frac{mv^2}{R} + Mg$

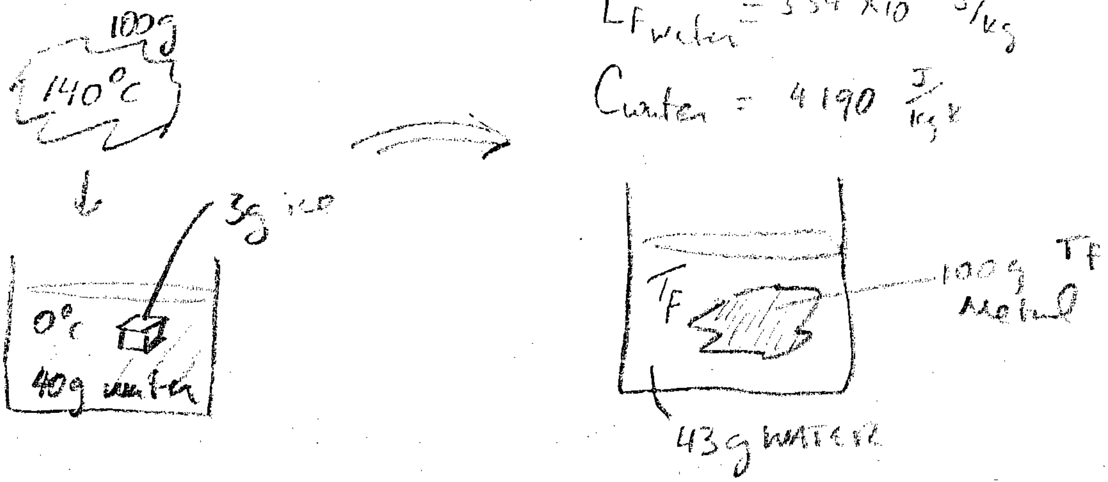
Fract change $= \frac{\frac{mv^2}{R} + Mg - Mg}{Mg} = +0.63$

Problem 5 (10 pts):

A 100 g chunk of metal is heated to 140 °C and is dropped into 40 g of water and 3 g of ice at 0 °C in a thermally insulated vessel. The final temperature of the mixture is 11.2 °C. What is the specific heat of the metal? Assume the process happens quickly and that no heat leaks away or into the system from the surrounding environment.

$L_{F\,water} = 334 \times 10^3 \ J/kg$

$C_{water} = 4190 \ \frac{J}{kg\,K}$



heat energy necessary to melt ice $= (334 \times 10^3 \ J/kg)(0.003) = 1002 \ J$

heat given up by Metal $= M_{metal} C_{metal} (140 - T_f) = (0.1 \ kg) \, C \, (140 - T_f)$

heat given up by Metal = heat to melt ice at 0°C + heat to warm water (43g) to T_f

$$0.1 \ kg \ C(140 - \underset{11.2}{T_F}) = 1002 \ J + (0.043 \ kg)\left(4190 \ \frac{J}{kg\,K}\right)(\underset{11.2}{T_F} - 0)$$

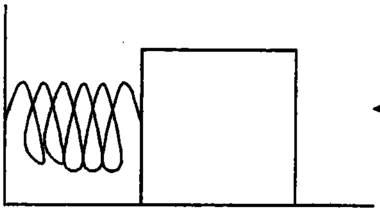
$$12.9 C = 1002 + 2018$$

$$\boxed{C = 234 \ \frac{J}{kg \cdot K}}$$

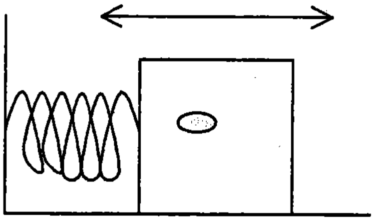
FYI happens to be silver

Problem 6 (10 pts):

Sinister Sam fired a rifle bullet into a block of wood with mass 5.0 kg attached to a spring on a frictionless table. The bullet, with a mass of 8.0 g, remains embedded in the block of wood. Initially, the block/spring are motionless. After the bullet is embedded the block oscillates back and forth along the table with amplitude 0.16 m and a period of 0.71 s. Calculate the velocity of the bullet just before it hits the block. 4.5



Before bullet hits wood



After bullet hits wood

P cons

$$m V_b = (M+m) V_{Bb}$$

V_b: Velocity of Bullet

V_{Bb}: Velocity of bullet + Block just after bullet is embedded

E cons After collision

$$\frac{1}{2}(m+M) V_{Bb}^2 = \frac{1}{2} k A^2$$

$$T = 2\pi \sqrt{\frac{m+M}{k}} \Rightarrow k = \left(\frac{2\pi}{T}\right)^2 (m+M) = \left(\frac{2\pi}{4.5}\right)^2 (5.008) = 9.8 \ N/m$$

use k + E cons to find V_{Bb}

Newton = kg M/s²

$$V_{Bb} = \sqrt{\frac{kA^2}{m+M}} = \sqrt{\frac{9.8 \ N/m \ (0.16)^2 m^2}{5.008 \ kg}} = 0.22 \ M/s$$

use V_{Bb} + P cons to get V_{bullet}

$$(0.008) V_b = (5.008)(0.22)$$

$$V_{bullet} = 138 \ M/s$$

Problem 7 (10 pts):
Short answer/multiple choice. No partial credit within each part.

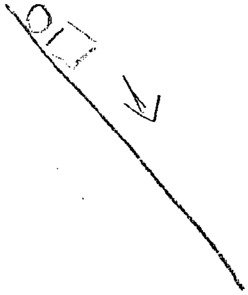
a) If the momentum of a particle is doubled, by what factor is the kinetic energy of that particle increased? (Your choices are factor of two, factor of one-half, factor of four, factor of one-quarter, or it remains unchanged.)

$P = mV$ If P doubled, V doubled

$KE = \frac{1}{2}mV^2$ If V doubled → KE mult by 4

FACTOR of 4

b) A cube and a marble of the same mass start from rest and slide down a frictionless inclined plane. Which object makes it to the bottom of the inclined plane first. (Your choices are the cube, the marble, or they reach the bottom at the same time.)

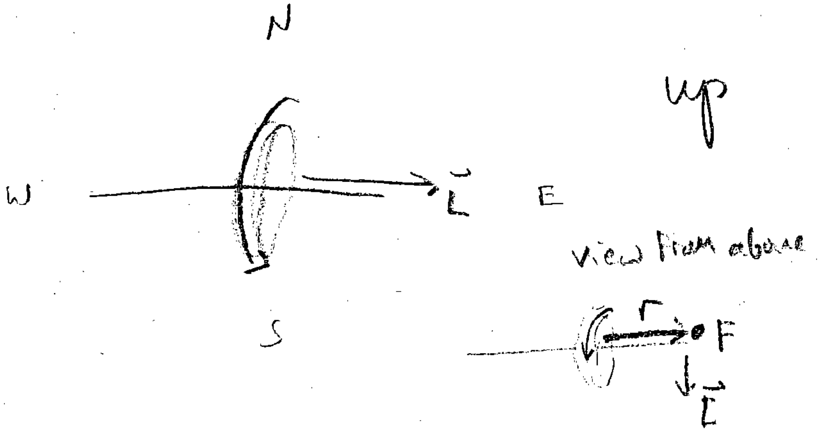


Inclined plane is frictionless

∴ Marble does NOT ROTATE

They reach bottom at the same time

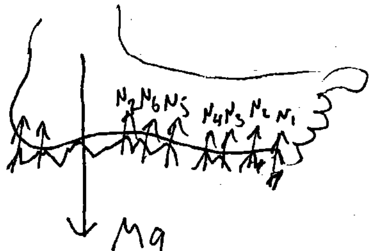
c) The angular momentum vector for a spinning wheel lies along its axle and is pointed east. To make this vector point south, it is necessary to exert a force on the east end of the axle in which direction? (Your choices are up, down, north, east, or south.)

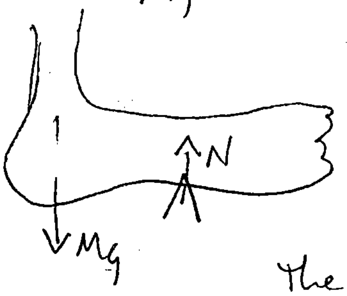


$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F}$

Problem 8 (10 pts):

a) A week and a half ago, many of you watched as one of your classmates danced on a bed of broken glass with bare feet. Explain why Trisha didn't end up with bloody cuts on her feet when she did this.





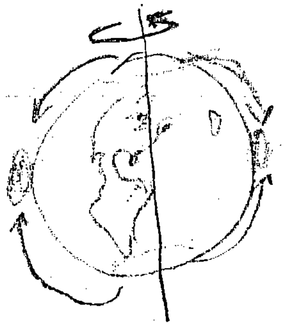
Generally ~~one~~ one would cut a foot on glass if the normal force of the jagged edge on the skin is large enough to penetrate. This happens when you step on a single piece of glass. In stepping on many chunks of glass, the $N = Mg$ force necessary to support Trisha's weight is spread over many points of contact. So the Normal force ~~to~~ at any single point of contact is too small to penetrate the skin.

(b) If global warming occurs over the next century, it is likely that some polar ice will melt and the water will be distributed closer to the equator. How would this change the length of a day (defined as the period for one rotation)? Explain your reasoning.

See next page

NOW [illegible] the spin [illegible]

(b) If global [illegible] occurs over the next century, it is likely that some po will be distributed closer to the equator. How would this change the length of a da for one rotation)? Explain your reasoning.



Polar ice melts -> moves to equator

increases I_{earth}

Angular Momentum is conserved

$$I_{NOW}\omega_{NOW} = I_{Future}\omega_{Future}$$

$$I_{future} > I_{NOW} \Rightarrow \omega_{future} < \omega_{NOW}$$

ω will become slightly smaller

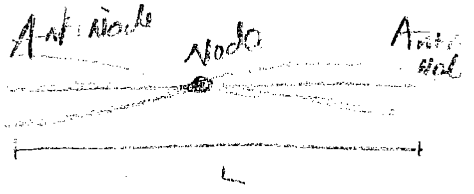
$\Rightarrow$ longer to make a single rotation

$\therefore$ the length of the day increases slightly

Problem 9 (10 pts):

One can hold a long, slender aluminum rod near the rod's midpoint and stroke the rod with the other hand and make the rod "sing", or emit a clear, loud, ringing noise.
Useful information: the speed of sound in aluminum is 5100 m/s. This is the speed of the waves traveling in the rod that cause the sound.

STANDING wave pattern



Fund. freq.

(a) For a 90 cm rod, calculate the fundamental frequency in the rod.

$L = \lambda_0/2$ $\qquad V = \lambda \nu$

$\nu = V/\lambda$

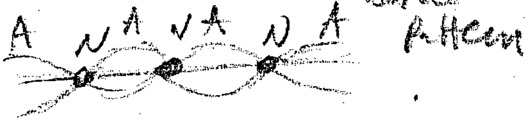
$L = \frac{V}{\nu_0 2}$ $\qquad \nu_0 = \frac{V}{2L} = \frac{5100}{2(0.9)} = \boxed{2833 \text{ Hz} = \nu_0}$

(b) For this 90 cm rod, what is the wavelength of the fundamental frequency?

$\boxed{\lambda = 2L = 1.8 M}$

(c) For this 90 cm rod, what is the frequency of the first harmonic?

1st harmonic STANDING wave pattern



$L = \frac{3}{2}\lambda_1$

$L = \frac{3}{2}\frac{V}{\nu_1}$ $\qquad \nu_1 = \frac{3}{2}\frac{V}{L} = \frac{3}{2}\frac{5100}{0.9}$

$\boxed{\nu_1 = 8500 \text{ Hz}}$

(d) Would you expect the fundamental frequency you hear to change with temperature? Explain why or why not.

$$\nu_0 = \frac{V}{2L}$$

L changes w/ temperature $\Rightarrow$ ν_0 changes

Also Might expect speed of sound to change in the metal this would also cause ν_0 to change via formula above

Problem 10 (10 pts):

A heavy rope with a constant mass per unit length μ and total length L hangs from the ceiling. A person of mass M hangs from the bottom of the rope and excites transverse wave pulses in the rope. Find and circle the correct expression for the velocity of the transverse waves on this rope as a function of distance, y, from the bottom of the rope. You must show your work to receive credit for this problem.

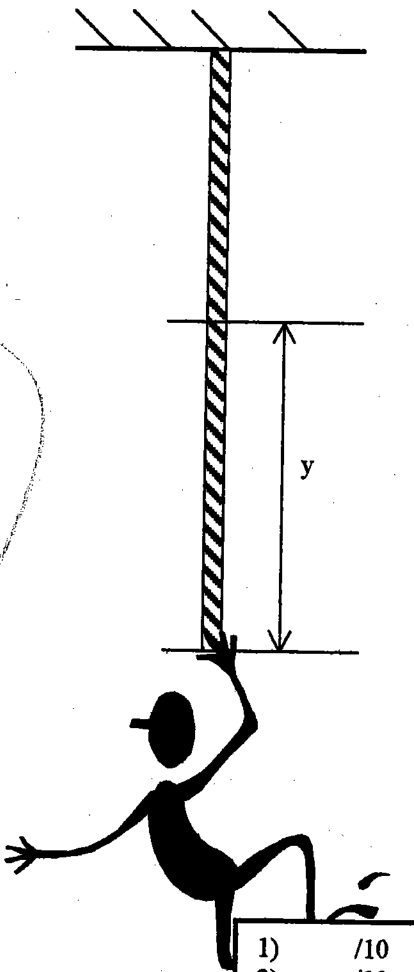
(a) $\sqrt{\frac{Mg}{\mu}}$

(b) $\sqrt{Lg}$

[✓ (c) $\sqrt{\frac{Mg}{\mu}+gy}$]

(d) $\sqrt{gy}$

(e) $\sqrt{\frac{Mg}{\mu}+gL}$



$V = \sqrt{\frac{T}{\mu}}$ for velocity of TRANSVERSE Wave on a String

what is $T(y)$?

$T(y) = Mg + \mu y g$

$\therefore V(y) = \sqrt{\frac{Mg + \mu y g}{\mu}}$

$\boxed{V(y) = \sqrt{\frac{Mg}{\mu} + gy}}$

1)	/10
2)	/10
3)	/10
4)	/10
5)	/10
6)	/10
7)	/10
8)	/10
9)	/10
10)	/10
tot	/100

We will grade the exams on Wednesday and Thursday of this week. It will take me a couple of days to calculate all the grades. It is doubtful I can finish before the weekend. I don't know actually know if the registrar will be open over the weekend or on Monday. If so, your grades will show up on Access then. If not, they won't show up until the registrar's office reopens after Xmas. If I do not run into technical difficulties, I will send you email summarizing your scores for the semester.

Have a wonderful holiday! Thrill you parents with a discussion about the standing waves in their eggnog!