

### Final Exam (December 18, 2000)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

#### Problem 1 (10 pts):

Jason Snodwartz, a very typical college physics student, decides to spend New Year's Day pondering the beautiful subtleties of classical Newtonian mechanics rather than watch football or cuddle with his (soon to be former) girlfriend. Before he curls up with a limited edition copy of "Vectors for Connoisseurs" by I.M. Adweeb, Jason makes a cup of cappuccino.

As part of the process, he starts with 50 grams of milk at 10 degrees centigrade and adds 10 grams of steam at 100 degrees centigrade. Calculate the final temperature of the milk-steam mix. Assume milk has the same heat capacity as water. Also assume the milk and steam are mixed in a cup that is thermally isolated from the rest of the universe.

$$C_{\text{milk}} = C_{\text{water}} = 4190 \frac{\text{J}}{\text{kg}\cdot\text{K}} \quad L_v = 2256 \times 10^3 \frac{\text{J}}{\text{kg}}$$

$$|Q_{\text{milk}}| = m_{\text{milk}} C_{\text{milk}} (T_f - 10) \quad \text{for } T_f = 100 \quad Q_{\text{milk}} = 1.89 \times 10^4 \text{ J}$$

$$|Q_{\text{steam} \rightarrow \text{water}}| = (0.01) (2256 \times 10^3) = 2.25 \times 10^4 \text{ J}$$

$\Rightarrow \therefore$  The heat given off by steam condensing to water is more than it takes to warm milk up to  $100^\circ\text{C}$

So steam condenses until milk is  $100^\circ\text{C}$

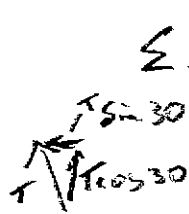
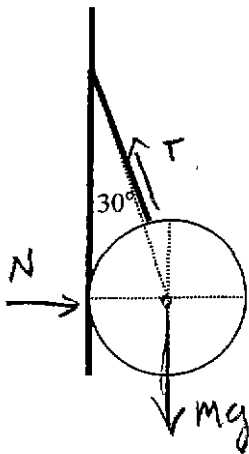
At that point thermal equilibrium is achieved at  $100^\circ\text{C}$  with a steam-milk mix

$\Rightarrow$  This was unintentional. I made an error when I made out this problem. It was intended to be more standard w/  $T_f$  between 0 and 100.

**Problem 2 (10 pts):**

Consider a uniform ball of radius 0.2 m and mass 0.75 kg held against a frictionless wall by a massless string as shown in the diagram below. The string makes an angle of 30 degrees with the wall.

a) Find the tension in the string and the normal force of the wall against the ball assuming the line joining the wall to the ball extends through the center of the ball (drawn imperfectly below).



$$\sum \vec{F} = 0 \Rightarrow \begin{aligned} \sum F_x = 0 & \quad N - T \sin 30 = 0 \\ \sum F_y = 0 & \quad Mg - T \cos 30 = 0 \end{aligned}$$

$$N = T \sin 30$$

$$Mg = T \cos 30 \rightarrow T = \frac{Mg}{\cos 30}$$

$$N = Mg \tan 30 = 4.2 \text{ Newtons}$$

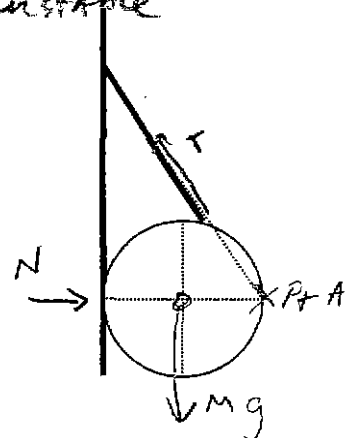
$$\frac{4.2}{\sin 30} = T = 8.4 \text{ Newtons}$$

No Need to use torque eqn

b) Is it possible for the configuration below to be in static equilibrium if the wall is frictionless? In this configuration the line joining the wall to the ball passes through the ball as shown. Why or why not?

Consider axis thru P/A. Mg gives an unbalanced Torque  
∴ There must be a torque to counterbalance this if system is in static equilibrium. Friction would provide such a torque. Without friction, system is unstable

c) Draw and label all the forces on the ball on the diagram below.

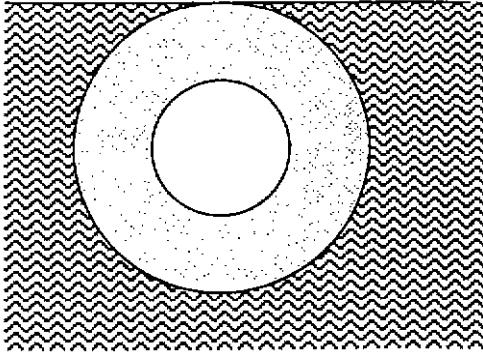


1)	/10
2)	/10
3)	/10
4)	/10
5)	/10
6)	/10
7)	/10
8)	/11
9)	/10
10)	/10
11)	/10
12)	/10

**Problem 3 (10 pts):**

Judy and Bill take a bath together ... now, now, now, I know what you're thinking. This has nothing to do with the physics of that one poster we saw last week. Judy and Bill are three years old. To them, Kama Sutra is the name of the newest Power Ranger.

Anyway, Judy and Bill have hollow, plastic, spherical ball in the bath. The ball floats in the bath water almost submerged. The outer radius is 0.1 m. The volume density of the plastic in the ball is  $2.0 \text{ g/cm}^3$ . Assuming the hollow space inside the ball is also spherical and centered in the ball (as shown below), what is the inner radius of the spherical shell, i.e., the outer radius of the inner hollow space?



$$mg = F_b = W_{\text{water displaced}}$$

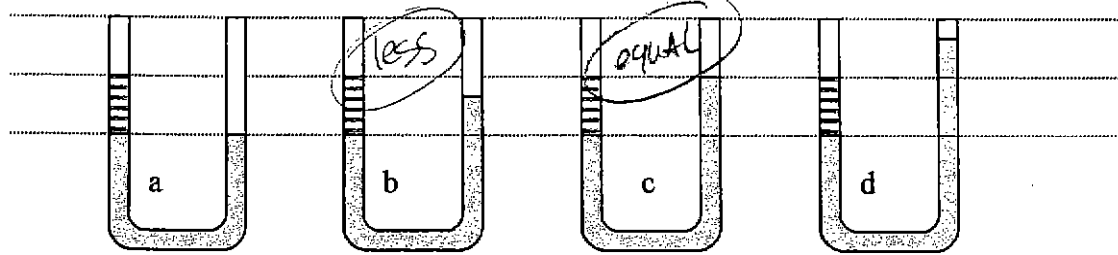
$$\rho_{\text{pl.}} V_{\text{pl.}} g = \rho_w g \frac{4}{3} \pi R_o^3$$

$$\rho_{\text{pl.}} \left( \frac{4}{3} \pi R_o^3 - \frac{4}{3} \pi R_i^3 \right) = \rho_w \frac{4}{3} \pi R_o^3$$

$$\rho_{\text{pl.}} R_o^3 - \rho_w R_o^3 = \rho_{\text{pl.}} R_i^3$$

$$R_o^3 \frac{(\rho_{\text{pl.}} - \rho_w)}{\rho_{\text{pl.}}} = R_i^3 = (0.1)^3 \frac{1}{2} \Rightarrow R_i = .081 \text{ m}$$

**Problem 4 (10 pts):**



In the glass tubes pictured above is a gray liquid and a different liquid portrayed with horizontal stripes.

a) Of the four choices, <sup>two</sup> ~~one~~ situation <sup>is</sup> ~~is~~ impossible. Which <sup>are they</sup> ~~is it?~~

(a) and (d)

b) In the other <sup>two</sup> ~~three~~ situations. Is the striped liquid density less than, equal to, or greater than the gray liquid? Indicate your answer on the diagram.

(b) less than

(c) equal to

tot /121

**Problem 5 (10 pts):**

Consider two identical masses that travel in circles of different radii on a frictionless surface. Each is tied to a string with the other end attached to a rod in the center of the circle. The strings can rotate freely about the peg. The mass/string configurations are shown below in a view from above.

a) If the two masses travel at the same speed, which string is more likely to break? Why?

Tension =  $\frac{mv^2}{R}$  <sup>Linear</sup>  
 m and v are the same  $\Rightarrow$  Smaller radius gives larger Tension  
 Smaller radius string more likely to break.

b) If the two masses have the same period, which string is more likely to break? Why?

$$v = \frac{2\pi R}{\text{Period}}$$

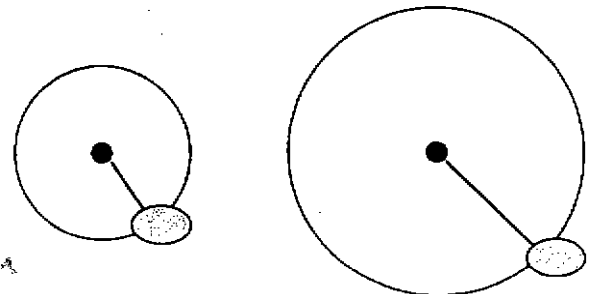
Same period implies  $\frac{v}{R} = \text{const}$  while

$$\text{Tension} = \left(\frac{mv}{R}\right)v$$

Same for both

v is larger for larger radius case

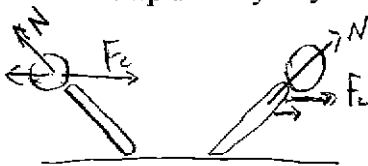
Tension goes as v  $\rightarrow$  larger for larger radius



**Problem 6 (10 pts):**

Explain why bicyclists and motorcyclists lean into turns.

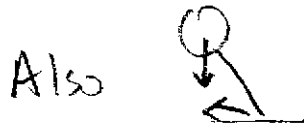
$\Rightarrow$  larger radius cool bikes



Provides a torque about point of tire contact to the road that causes  $\vec{I}$  of wheel to precess in the direction of turn.

Leaning also orients the normal force of seat on person such that it contributes toward centripetal force on person. Otherwise seat must provide large transverse force on person. So that total transverse force is enough to move person with bike. If not, person falls.

best

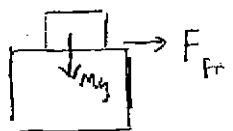


Also leaning means torques due to c. mass and friction could wheel when turned

**Problem 7 (10 pts):**

Consider a 2 kg mass stacked on top of a 7 kg mass as shown below. The two masses are attached to a spring (which is attached to the wall) and can move back and forth on a frictionless surface. The coefficient of static friction between the bottom surface of the top mass and the top surface of the bottom mass is 0.45. The spring constant is 200 N/m.

- a) What is the maximum amplitude of the simple harmonic motion of the system shown below such that the top mass does not slip during the oscillation?



For no slipping,  $a_{\text{small block}} = a_{\text{big + little block system}}$   
from friction  from force of spring

MAX Amplitude =  $A$

$$m_{\text{small}} g \mu_s = kA$$

$$(2)(9.8)(0.45) = 200A \quad A = 0.044 \text{ m}$$

- b) In a system that undergoes this limiting motion: What is the period?

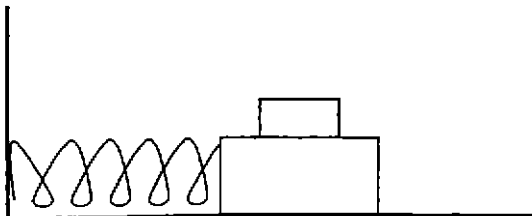
$$T = 2\pi \sqrt{\frac{M}{k}} = 2\pi \sqrt{\frac{7+2}{200}} = 1.33 \text{ s}$$

- c) In a system that undergoes this limiting motion: What is the frequency?

$$f = \frac{1}{T} = \frac{1}{1.33} \text{ Hz} = 0.75 \text{ Hz}$$

- d) In a system that undergoes this limiting motion: What is the total energy of the system?

$$E_{\text{TOT}} = \frac{1}{2} k A^2 = \frac{1}{2} (200) (0.044)^2 = 0.19 \text{ J}$$



**Problem 8 (11 pts):**

Consider four forces of equal magnitude  $F$  acting on a block that slides on a frictionless table as pictured below. The block slides, presumably due to some other force (not shown), a distance  $x$  on the table from left to right.

a) How much work is done by each of the forces shown?

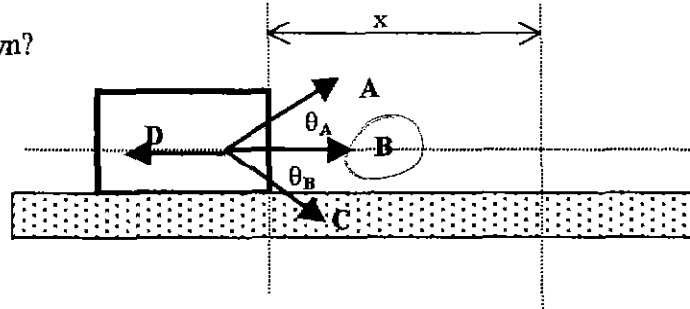
$$W = \int F \cdot dx$$

$$W_A = F \cos \theta_A x$$

$$W_B = Fx$$

$$W_C = F \cos \theta_B x$$

$$W_D = -Fx$$



b) Circle the force that does the most work as the block.

B

Consider five forces of equal magnitude acting on a wheel with moment of inertia  $I$  as shown below. (Force D is coming out of the paper.)

c) Circle the force that produces the largest torque on the wheel..

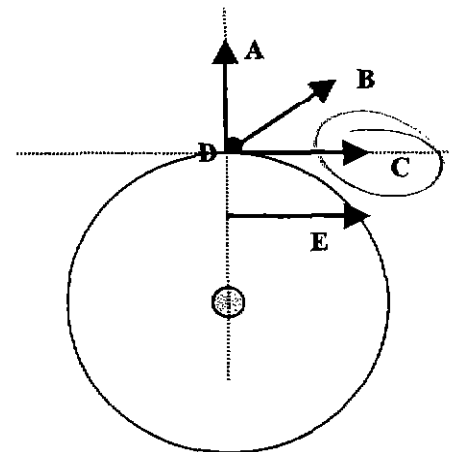
C

d) What is the direction of the torque about the wheel axis (if any) produced by each of the forces below acting on the wheel?

A: NO Torque abt Axis

D: NO Torque abt Axis

B; C, E all produce torque into paper  
 causing wheel to rotate  $\curvearrowright$



**Problem 9 (10 pts):**

After a big basketball game, Leslie "The Jammer" Johnson's agent stops the famous ball player at mid-court and tells him that he will only make \$25 million during the following season. A distraught Jammer Johnson stands still in the middle of the court and ponders how he's going to manage to make ends meet on such a paltry salary. The crowd leaves. The janitor and arena crew come in and polish the court, working around the stunned ball player. However, they overdo it. By the time Jammer decides to move on with his life, he finds that the court is so slick around him that he can't walk on it. Like most pro basketball players, Jammer Johnson is an avid student of physics. So, he pulls off his shoes that weigh 3 kg (big feet, you know) and throws them at 10 m/s toward one side of the court.

- a) What happens to Jammer when he throws the shoes (assuming the floor to be frictionless)? Why?

Momentum conservation implies that Jammer must slide across the floor in a direction opposite his thrown shoes.

Also could discuss Newton's 3<sup>rd</sup> law Action = Reaction

- b) Assuming Jammer weighs 85 kg, how long does it take before Jammer reaches the nearest wall 20 meters away?

$$M_J V_J + M_{shoes} V_{shoes} = 0$$

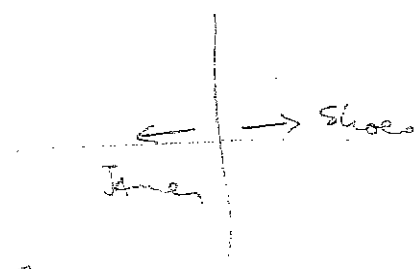
$$3 \text{ kg} (10 \text{ m/s}) = -85 \text{ kg} V_J$$

$$V_J = 0.35 \text{ m/s} \text{ opposite shoes}$$

$$\Delta s = v \Delta t$$

$$20 = 0.35 \Delta t$$

$$\Delta t = 57.5$$



**Problem 10 (10 pts):**

A dog, is standing on a flat boat so that he is 15 m from the shore. He walks 3 m on the boat toward shore and then halts. One can assume there is no friction between the boat and the water.

a) What happens to the boat as the dog walks toward the shore? Why?

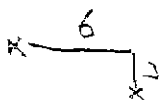
The Center of Mass of the boat does not move.  
∴ The boat must move away from the shore to offset dog's movement



b) What happens to the center-of-mass of the boat-dog system? Why?

Does NOT move. No external forces on the dog-boat system

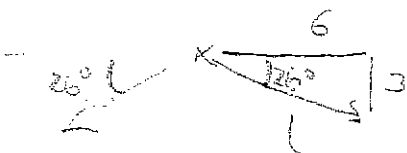
c) Suppose the dog jumps into the water exactly 15 m from shore. He swims 6 meters parallel to the shore and then he swims 3 meters toward the shore. How close is the dog to the shore now?



Now 12 m from shore



d) Considering the situation in part (c), what is the dog's total displacement vector from the moment he started swimming?



Magnitude = 6.7 m

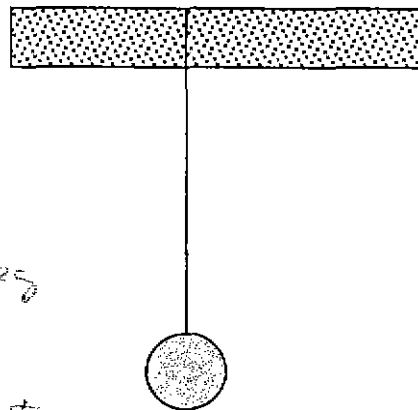
Shore

direction is 26° toward shore from parallel line out. Unable to tell whether it is to left or right from into given



**Problem 11 (10 pts):**

Thog the Hairy was the world's first musician. Neanderthal lore says his first instrument was made from a 5 kg rock hanging from a 2 m long mammoth-hair string with a mass per unit length of 1.5 g/cm tied to a tree. What were the frequencies of the lowest 3 harmonics Thog could play on this instrument? (Assume the rock is heavy enough that its transverse motion is negligible.)



Tension in string =  $M_{\text{Rock}} g$

Neglecting Mass of String

String mass

$2 \text{ m} \times 1.5 \text{ g/cm} \times 100 \frac{\text{cm}}{\text{m}} = 300 \text{ g} = 0.3 \text{ kg}$

$0.3 < 5.0 \rightarrow$  Accept

Answer part

Neglect Mass of String

$T = (3.0)(9.8) = 49 \text{ Newtons}$

$V_{\text{waves on string}} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{0.15}}$

$\mu = 1.5 \frac{\text{g}}{\text{cm}} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times 100 \frac{\text{cm}}{\text{m}} = 0.15 \frac{\text{kg}}{\text{m}}$

$\mu = 0.15 \text{ kg/m}$

both ends fixed

$V = 326.7 \text{ m/s}$

$\frac{\lambda_1}{2} = L$  (n=1)

$\lambda_2 = L$

$\frac{3}{2} \lambda_3 = L$

$L = \frac{n}{2} \lambda_n \quad n = 1, 2, 3, \dots$

$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, \dots$

$v = \lambda v$

$\frac{v}{\lambda_n} = v_n$

$\frac{v n}{2L} = v_n = \frac{326.7}{2} n = v_n$   
(2x2)

$v_1 = 8.1 \text{ Hz}$

$v_2 = (2) 8.1 = 16.2 \text{ Hz}$

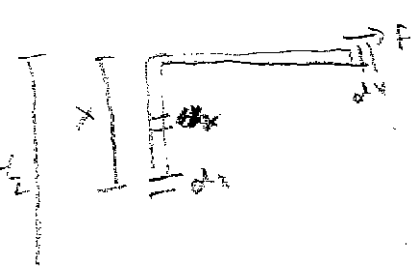
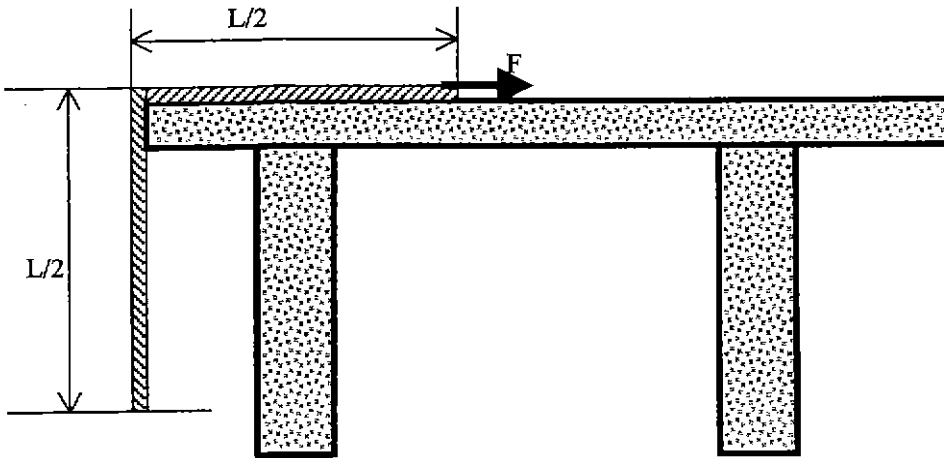
$v_3 = (3) 8.1 = 24.3 \text{ Hz}$

NOTE: Must be in Hz

**Problem 12 (10 pts):**

Consider the configuration (shown below) of a rope that is half hanging over the edge of a frictionless table. Suppose the rope has a mass per unit length of  $\lambda$  kg/m. How much work is exerted by a force that slowly pulls the entire rope up onto the table? Select the correct answer and show the work that led you to that answer. No credit will be given for correct answers if there is no supporting proof.

- a)  $L^2 g \lambda$    b)  $\sqrt{gL}$    c)  $\frac{L^2 g \lambda}{8}$    d)  $\frac{L^2 g \lambda}{4}$    e)  $\sqrt{2gL}$



$$W = \int F \cdot dx$$

$$W = \int_0^{L/2} x \lambda g dx = \frac{x^2 \lambda g}{2} \Big|_0^{L/2}$$

$$F_{\text{inst}} = x \lambda g$$

Mass of rope hanging over  
 Table edge

$$W = \left(\frac{L}{2}\right)^2 \frac{1}{2} \lambda g = \frac{L^2}{8} \lambda g$$

**Have a wonderful holiday!**

The dean says I'm not allowed to post grades. I plan to determine grades by the end of the week. I hope to send them to you individually via e-mail around then. That involves some work on my part ... the e-mail could slip until after New Year's Day.