P121 University of Rochester NAME Solution key SM
S. Manly Spring 2001

Put in vector Arrows

Exam 3 (April 12, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (20 pts):

Consider vector $\vec{A} = 6i + 0j + 0k$ and vector $\vec{B} = 3i - 2j + 0k$.

(a) What is the angle between the vectors?

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \rightarrow$ $18 = \sqrt{13}(6)\cos\theta$

$\theta = 33.7°$

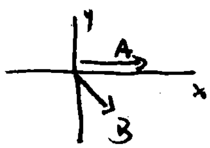
$\vec{A}\cdot\vec{B} = (6)(3) = 18$

$|\vec{A}| = 6 \qquad |\vec{B}| = \sqrt{9+4} = \sqrt{13}$

(b) What is the magnitude of $\vec{A}x\vec{B}$? If you don't get an answer for part A, use a symbol for the angle and evaluate B in terms of that angle.

$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta = \sqrt{13}(6)\sin(33.7) = 12$

(c) What is the direction of $\vec{A}x\vec{B}$?

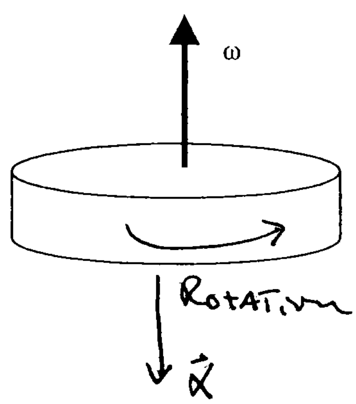


$\vec{A}\times\vec{B}$ into paper along $-z$ axis or $-\hat{k}$

Consider the spinning wheel drawn below. The angular velocity vector is shown.

(d) Indicate on the drawing the direction of rotation.

(e) The spinning wheel is slowing down with time. Draw the angular acceleration vector on the sketch, assuming any nonzero magnitude you wish.



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Problem 2 (20 pts):

At the zoo, a monkey hangs from a 3-m long, 18 kg uniform rod that is held in place by ropes at its ends. The left rope makes an angle of 150 degrees with the rod and the right rope makes and angle θ with the horizontal. The monkey weighs 10 kg and hangs motionless 0.5 meters from the right end of the rod. Determine the tensions in the two ropes and the angle θ.

1)	/20
2)	/20
3)	/20
4)	/20
5)	/20
6)	/20
tot	/120

I $\sum F_x = -T_1 \cos 30 + T_2 \cos\theta = 0$

II $\sum F_y = T_1 \sin 30 + T_2 \sin\theta - m_b g - m_m g = 0$

III $\sum \vec{\tau} = 0 = (1.5) m_b g + 2.5 M_m g - (3) T_2 \sin\theta = 0$

III $T_2 \sin\theta = \dfrac{(1.5)(18)(9.8) + (2.5)(10)(9.8)}{3} = 169.8\ N$

sub in II $T_1 \sin 30 = (18)9.8 + (10)(9.8) - 169.9 \Rightarrow \boxed{T_1 = 209\ N}$

I $\rightarrow$ $T_1 \cos 30 = T_2 \cos\theta$

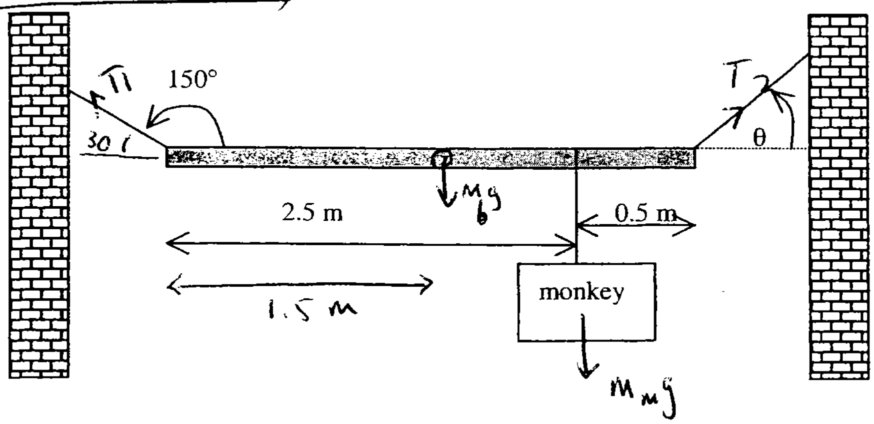
$209 \cos 30 = T_2 \cos\theta$

II $\rightarrow -T_1 \sin 30 + m_b g + M_m g = T_2 \sin\theta$

$\tan\theta = \dfrac{(M_b + M_m)g - T_1 \sin 30}{T_1 \cos 30}$ (18, 10, 9.8, 209; 209)

$\boxed{\theta = 43.2^\circ}$

$\dfrac{T_1 \cos 30}{\cos 43.2} = \boxed{T_2 = 248.3\ N}$



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Problem 3 (20 pts):

(a) Two eggs, identical in mass, shape and size, roll without slipping down an inclined plane. One egg is raw and the other is hard-boiled. Which egg will make it to the bottom first? Justify your answer.

at the bottom the Δ in grav. PE is the same for both eggs. This energy is divided into KE of TRANSlation and KE of ROTATION.

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

The larger I happens to be, the smaller is the energy available for TRANSlational motion. For a Hard boiled egg, all of the egg mass rotates with the egg. For a raw egg, the liquid inside ~~does not rotate~~ is NOT constrained to rotate with the outside of the egg. ∴ $I_{\text{raw egg}} < I_{\text{cooked egg}}$

∴ The $V_{\text{raw egg}} > V_{\text{cooked egg}}$ ⇒ The raw egg reaches the bottom first!

(b) A gymnast is under instructions from her coach to spin her body around as many times as possible in the forward direction upon her dismount from the balance beam. Should she keep her body straight or tuck her knees tightly to her body? Why?

Angular momentum conservation for an isolated system, such as the spinning gymnast, implies that $I\omega$ is CONSTANT.
If the gymnast tucks her knees tightly to her body, she will reduce her overall moment of Inertia ($\sim mR^2$... she reduces R for a substantial part her mass). Thus her angular velocity of rotation will increase (because $L = I\omega$ is CONSTANT). Increasing ω means more rotations in a given time!

could also say
raw egg has shifted C.M. due to yolk dropping a little
+ make an argument

0

Essential → must relate I difference to V difference

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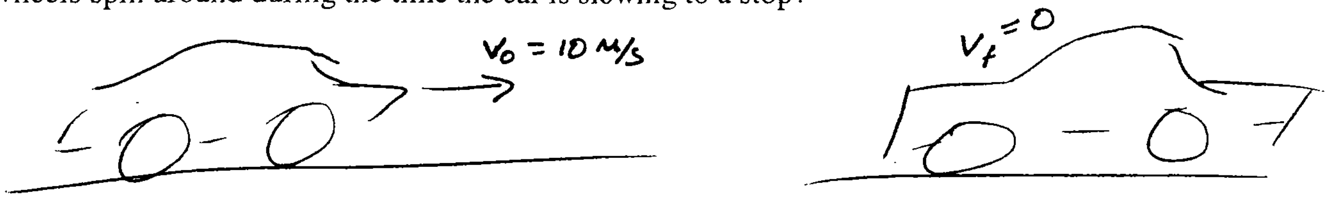
Problem 4 (20 pts):

A car with an initial speed of 10 m/s slows down to a stop in 5 seconds with uniform acceleration (deceleration). The wheels each have a radius of 0.3 m. How times do the wheels spin around during the time the car is slowing to a stop?



$v = v_0 + at$

$0 = 10 + at$

$s = r\theta$

$v = r\omega$

$a = r\alpha$

Angular Soln

$\frac{-10}{5} = a = -\cancel{2}\ m/s^2$

if $a = -\cancel{2}\ m/s^2 \Rightarrow \alpha = \frac{-\cancel{2}\ m/s^2}{.3} = \cancel{-6.67}\ -6\tfrac{2}{3}\ rad/s^2 \rightarrow 6.667$

$\omega_0 = \frac{10}{.3} = 33.3\ rad/s$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$(\theta - \theta_0) = 33.3(5) - \frac{1}{2}(\cancel{1.67}\ 6.667)\,5^2 = \cancel{164.4}\ 84.2$

$\frac{84.2\ \cancel{164.4}\ rad}{2\pi} = \cancel{26.2}\ 13.4$ rotations

Linear Soln to check

$x - x_0 = v_0 t - \frac{1}{2}\cancel{2}(25)$
$(10)5$

distance (linear) = 25 m

circumf. = $2\pi r$
$= 2\pi .3$

$\frac{25}{2\pi(.3)} = 13.3$

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Problem 5 (20 pts):

A 5 kg mass is attached to a massless rope that passes without slipping across two pulley wheels, A and B. The rope is pulled with a tension of 150 N. Pulley wheel A has a mass of 2 kg and a radius of 0.3 m. Pulley wheel B has a radius of 0.15 m. Consider each pulley wheel as a solid, uniform cylinder. The 5 kg mass moves upward with an acceleration of 2 m/s2. Calculate the mass of pulley wheel B.

$\frac{1}{2}MR^2 = I_{wheel}$

$s = r\theta$
$a = r\alpha$

$I_A = \frac{1}{2}M_A R_A^2$
$= \frac{1}{2}(2)(.3)^2 = .09 \; k \cdot m^2$

3 eqns
unknowns T_1, T_2
I_B

I $\sum F_{mass} = ma = T_1 - mg$

II $\sum \tau_A = I_A \alpha_A = T_2 R_A - T_1 R_A$ $\alpha_A = \frac{a}{R_A}$

III $\sum \tau_B = I_B \alpha_B = T R_B - T_2 R_B$ $\alpha_B = a/R_B$

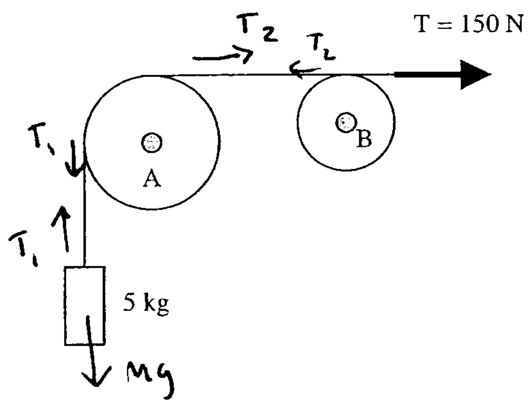
I_B → get this → can solve for $M_B = \frac{I_B}{R_B^2} 2$

I → $T_1 = m(a+g) = 5(2+9.8) = 59$ N

II → $T_2 = \frac{I_A \alpha_A + T_1 R_A}{R_A} = \frac{(.09)\frac{2}{.3} + (59).3}{.3} = 61$ N

III → $I_B = \frac{(T R_B - T_2 R_B) R_B}{a}$

$I_B = \frac{[(150)(.15) - (61)(.15)] .15}{2} = 1.001$



$I_B = \frac{1}{2} M_B R_B^2$

$1.001 = \frac{1}{2} M_B (.15)^2$

$M_B = 89$ kg

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Problem 6 (20 pts):

A 2.0 kg sphere attached to an axle by a spring is displaced from its rest position to a radius of 20 cm from the axle centerline by a mass of 20 kg, as shown in the drawing below on the left. The same 2 kg sphere is also displaced 20 cm from the axle centerline, as shown in the drawing below on the right, when the sphere is rotated (the axle rotates). Calculate the magnitude of the angular velocity of rotation of the sphere about the axis in the latter case (figure on the right).

left

$20g = K(.2)$

$mg = F_s$

$K = \frac{(20)(9.8)}{.2}$

$K = 980\ N/m$

Right hand

~~Balance between~~ centripetal Accel. supplied by Spring

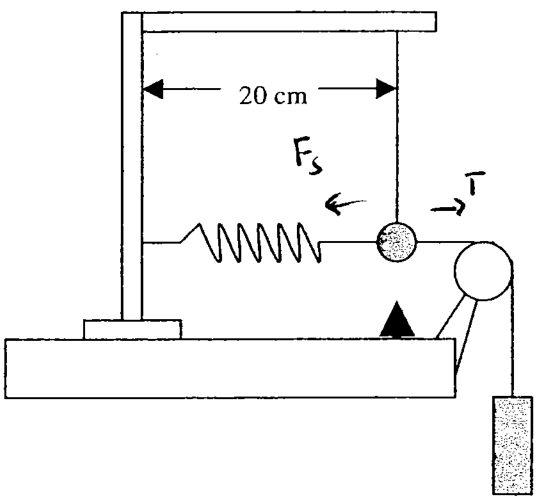
$\frac{Mv^2}{R} = 20g$

$v^2 = \frac{20(9.8)(.2)}{2}$ $V = 4.43\ m/s$

$s = r\theta$
$v = r\omega$

$\omega = \frac{v}{R}$

$\boxed{\omega = \frac{4.43}{.2} = 22.1\ rad/s}$



20 kg