P142 University of Rochester NAME Solution key - SM
S. Manly Fall 2010

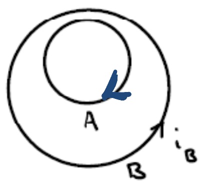
Exam 2 (October 16, 2010)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

<u>Problem 1 (9 pts, briefly indicate reasoning):</u>

In the three cases shown below the current i_B is increasing with time. Indicate the direction of the induced current in the loop labeled A in each case. If there is no current induced in loop A, indicate that by writing "no current" beside loop A for that case.

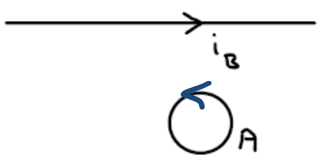
a)



Both loops in plane of paper

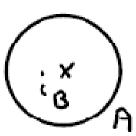
Clockwise
Want induced current in A to create $\vec{B}$ in direction opposite to that of increasing $\vec{B}$ from loop B

b)



Counterclockwise
Lenz's law - oppose $\vec{B}$ increasing from increasing i_B in wire.

c)

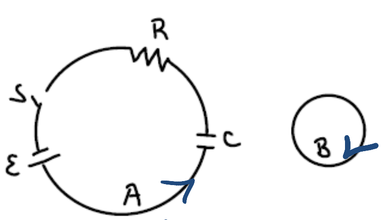


i_B into paper along axis of circular loop A

No induced current
$d\Phi_m/dt = 0$

<u>Problem 2 (9 pts, show your work):</u>

The sketch below shows an EMF arranged in series with a resister, R, and a capacitor, C. Qualitatively describe the current in loop B (which is in the same plane as loop A) as a function of time after switch S is closed in loop A. A graphical description is acceptable.

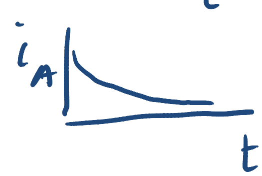


direction of changing current

as capacitor charges

$$Q = C\mathcal{E}(1 - e^{-t/\tau})$$

$$\left.\frac{dQ}{dt}\right|_A = \frac{C\mathcal{E}}{\tau}(e^{-t/\tau})$$



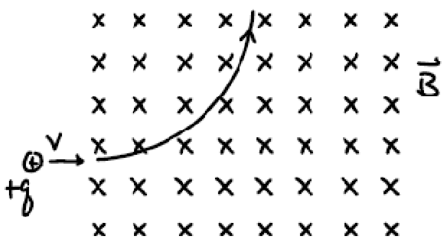
$\Phi_M|_B$ goes as i_A

$\mathcal{E}_A$ goes as $\Phi_M \sim i_A$

$\vec{B}$ in B due to i_A is into paper and weakening exponentially. So i_B clockwise and dropping exponentially in strength

Problem 3 (18 pts, use care, no partial credit for each part, no need to show work):

A charged particle moving at a constant speed enters a region of constant magnetic field as shown in the sketch below.



A number of changes to this initial situation are described in parts a-f below. Relative to this initial situation, select from choices i-v below that describe how each change will affect the magnetic force on the particle shortly after it enters the magnetic field. Place the best choice (i, ii, iii, iv or v) next to the part where the change is described.

i) This change will alter only the direction of the force on the particle.
ii) This change will only increase the magnitude of the magnetic force on the particle.
iii) This change will only decrease the magnitude of the magnetic force on the particle.
iv) This change will alter both the magnitude and direction of the magnetic force on the particle.
v) This change will not affect the magnetic force on the particle.

1)	/9
2)	/9
3)	/18
4)	/10
5)	/10
6)	/10
7)	/17
8)	/17
tot	/100

Each change below refers to the initial situation described above:

a) ii The +q particle is replaced by a +2q particle.

b) i The +q particle is replaced by a –q particle.

c) iv The +q particle is replaced by a neutral particle.

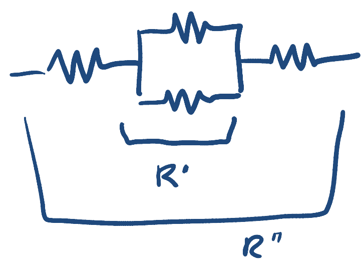
d) iii The particle enters the region moving at a slower initial velocity.

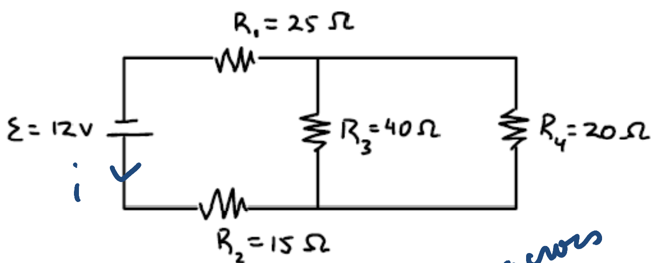
e) iii The magnetic field is one-third its original strength.

f) iv The direction of the magnetic field is parallel to the particle's initial velocity.

Problem 4 (10 pts, show your work):

Consider the circuit below. Determine the current in R_3.





$\frac{1}{R'} = \frac{1}{R_3} + \frac{1}{R_4} \rightarrow R' = 13.3\ \Omega$

$R'' = 13.3 + 25 + 15 = 53.3$

current in circuit

$12 = iR'' = i\,53.3 \quad i = 0.23A$

v drop across R_3

$V_{R'} = iR'$

$V_{R'} = (0.23)\,13.3$

$V_{R'} = 3.06 \rightarrow V_{R'} = i_3 R_3 \quad 3.06 = i_3\,40 \quad i_3 = .08\,A$

Problem 5 (10 pts, show your work):

A cubical volume with sides of length 1 m contains 1 coulomb of electric charge, evenly distributed. What will be the charge density in the volume as observed by a scientist in a spaceship traveling past the volume at 0.95c. Assume the direction of travel for the spaceship is perpendicular to one of the faces of the cube of charge.

$\rho = Q/m^3 \quad \rho' = \gamma Q/m^3$

Amount of charge unaffected

dimension of cube parallel to direction of spaceship travel is length contracted. ρ increased by factor of γ

$\gamma = \frac{1}{\sqrt{1-.95^2}} = 3.2$

$\rho' = 3.2\ \text{coul}/m^3$

Problem 6 (10 pts, show your work):

(a) What is the energy per unit length stored in an infinite solenoid of radius R_1, n turns per unit length and carrying a current I?

$B = \mu_0 i n$

$u_B = \frac{B^2}{2\mu_0}$

$u_0 = \frac{\mu_0^2 i^2 n^2}{2\mu_0} = \frac{\mu_0 i^2 n^2}{2}$

$\frac{U}{\ell} = \frac{\pi R_1^2 \mu_0 i^2 n^2}{2}$

(b) What is the energy per unit length if the same solenoid is filled with a paramagnetic material of permeability μ?

$B_{mat} = \chi B_{free}$

$\mu = \mu_0(1+\chi)$

$\frac{\mu}{\mu_0} - 1 = \chi$

$\frac{\mu}{\mu_0} > 1$

$B \rightarrow \left(\frac{\mu}{\mu_0} - 1\right)\mu_0 i n$

$u \rightarrow \left(\frac{\mu}{\mu_0} - 1\right)^2 \frac{\mu_0 i^2 n^2}{2}$

$\frac{U}{\ell} = \frac{\pi R_1^2 \mu_0 i^2 n^2}{2}\left(\frac{\mu}{\mu_0} - 1\right)^2$

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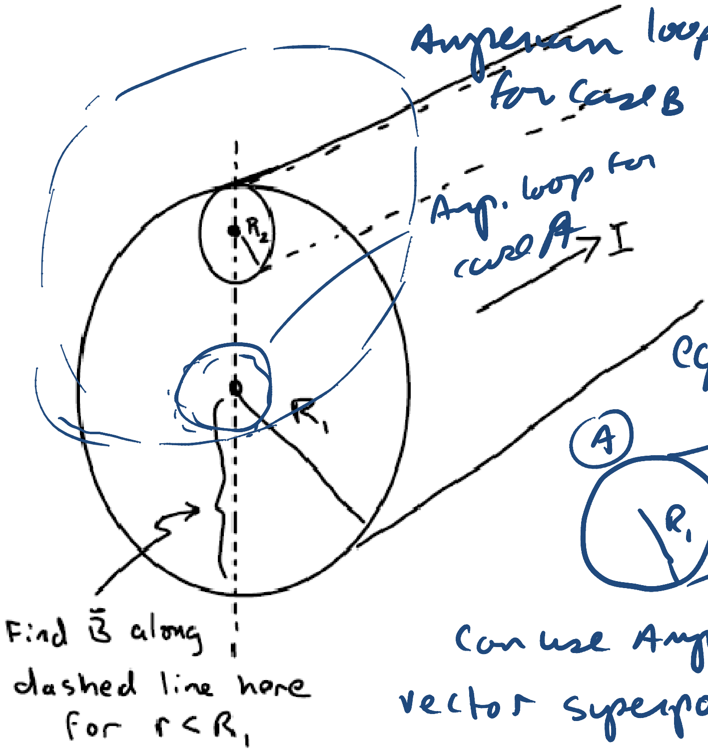
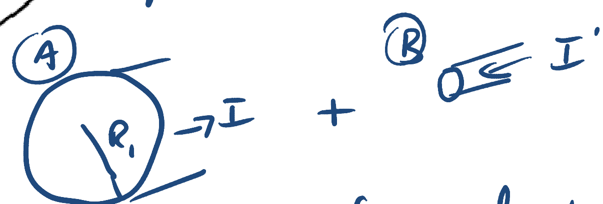
Problem 7 (17 pts, show your work):

Consider the infinite cylindrical conductor or radius R_1 sketched below. This conductor has a cylindrical hole of radius $R_2<R_1/2$ along its length. The hole is adjacent to the surface as shown. The conductor carries a current I with a uniform current density. Determine the magnetic field at a point along the line joining the center of the conductor and the center of the hole, on the for $r<R_1$ on the side opposite the hole.



lack of symmetry prevents use of ampere's law on geometry as drawn. So Assume geometry to left is equivalent to situation with



Can use Ampere's law for each + do vector superposition of two B fields.

Find $|\vec{j}| \rightarrow = \dfrac{I}{\pi R_1^2 - \pi R_2^2}$

Find $\vec{B}$ at $r<R_1$ on line below center for case A

$\int \vec{B}\cdot d\vec{l} = \mu_0 i_{encl}$ $\quad B_A 2\pi r = \mu_0 \pi r^2 j$ $\quad \vec{B}_A = \dfrac{\mu_0 r j}{2}$ clockwise

Find $\vec{B}$ at $r<R_1$ on line below center for case B

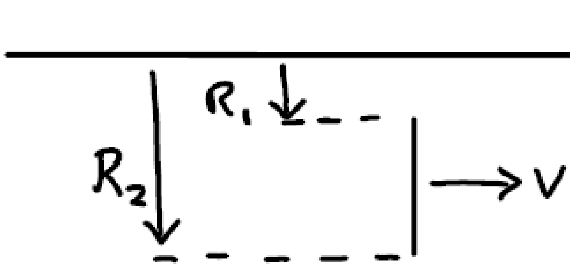
$\int \vec{B}\cdot d\vec{l} = \mu_0 i_{encl}$ $\quad B_B 2\pi(R_1+r-R_2) = \mu_0 \pi R_2^2 j$

$$\vec{B}_B = \frac{\mu_0 R_2^2}{2(R_1+r-R_2)} j \quad \text{counterclockwise}$$

$$\vec{B} = \frac{\mu_0}{2}\frac{I}{(\pi R_1^2 - \pi R_2^2)}\left[r - \frac{R_2^2}{R_1+r-R_2}\right]$$ where positive means clockwise

Problem 8 (17 pts, show your work):

An infinite straight wire carries current I. A thin, straight length of conductor moves in the direction along the wire at speed v as shown in the sketch. The moving conductor is perpendicular to the wire and oriented along a radial line from the current-carrying wire. Determine the potential difference between the two ends of the moving conductor.



$\vec{B}$ around wire at radius r

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$

Force on charge at radius $R_1 < r < R_2$ in conductor

$F = qvB = qv \frac{\mu_0 I}{2\pi r}$

$\frac{W}{q}$ to move q from R_1 to $R_2 = \Delta V_{ends} = \int_{R_1}^{R_2} \frac{\vec{F} \cdot d\vec{r}}{q}$

$$\Delta V_{ends} = \int_{R_1}^{R_2} \frac{v \mu_0 I}{2\pi r} dr = \frac{v \mu_0 I}{2\pi} \int_{R_1}^{R_2} \frac{1}{r} dr = \frac{v \mu_0 I}{2\pi} \ln \left| \frac{R_2}{R_1} \right|$$

Exam 2 Formulas

$\vec{F} = q\vec{E}$

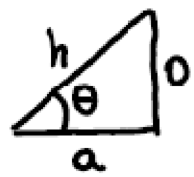
$|e| = 1.6\times10^{-19}$ coulombs

$k = 8.99\times10^{9}\ \frac{Nm^2}{C^2}$

$\vec{F} = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$

$\epsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$

$\phi_E = \oint \vec{E}\cdot d\vec{A}$



$\sin\theta = \frac{o}{h}\quad \cos\theta = \frac{a}{h}$

$\tan\theta = o/a$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$

Sphere: $A = 4\pi r^2\quad V = \frac{4}{3}\pi r^3$

$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\hat{r}$

cylinder: $A = 2\pi r L + 2\pi r^2$

$V = \pi r^2 L$

V = work/charge

$x' = \gamma(x - vt)$

$U'_x = \frac{U_x - V}{1 - \frac{V}{c^2}U_x}$

$V_{\text{point charge}} = \frac{kQ}{r}$

$y' = y$

$z' = z$

$V = \int_{vol}\frac{k\,dQ}{r}$

$t' = \gamma\left(t - \frac{Vx}{c^2}\right)$

$U'_{y,z} = \frac{U_{y,z}}{\gamma\left(1 - U_x\frac{V}{c^2}\right)}$

$\gamma = \frac{1}{\sqrt{1-\left(\frac{V}{c}\right)^2}}$

$E_s = -dV/ds$

$E = \gamma mc^2$

$p = m\eta = m\frac{dx}{d\tau} = m\gamma v$

$U_{capacitor} = \frac{1}{2} C V^2$

$Q = CV$

$E_{\parallel plate} = \sigma / \epsilon_0$

$\mathcal{U}_E = \frac{\epsilon_0}{2} E^2$

$P = iV = i^2 R = V^2/R$

$V = iR$

$F = q \vec{v} \times \vec{B}$

$|\vec{\mu}| = |niA|$

$\vec{\tau} = \vec{\mu} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$B_{solenoid} = \mu_0 n i$

$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$

$Q = C\mathcal{E}(1 - e^{-t/RC})$

$Q = Q_0 e^{-t/RC}$

$E = E_0 / K$

$\mathcal{E} = -d\Phi_M / dt$

$\Phi_M = \oint \vec{B} \cdot d\vec{a}$

$\mathcal{E} = -L \, di/dt$

$\Phi = Li$

$\mathcal{U}_B = B^2 / 2\mu_0 \qquad \mu = \mu_0 (1 + \chi)$

$B_{matter} = \chi B_{free}$

$\int u^n du = \frac{u^{n+1}}{n+1}$

$\int \frac{du}{u} = \ln |u|$

$\int e^u du = e^u$

$\int \frac{x\,dx}{(x^2 + a^2)^{1/2}} = \sqrt{x^2 + a^2}$