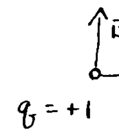
Exam 2 (March, 1999)

Please read the problems carefully and answer them in the space provided. Show all your work. Partial credit will be given (for all but problem 1).

Problem 1 (3 pts per part, 24 points total):

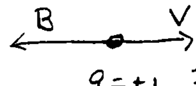
For each part below, state the *direction* of the instantaneous force of the magnetic field on the moving charge. The instantaneous position and velocity vector are shown. Assume current and magnetic field vectors are constant in each part. Your answer should only consist of one of the following: *into paper, out of paper, left, right, up, down, zero force exerted.* Do not bother to justify your answers. For this problem the answer for each part is either correct or incorrect. No partial credit will be given.

(a)



OUT OF PAPER

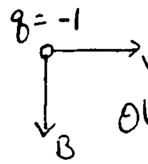
(b)



Zero Force

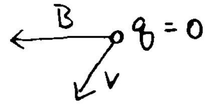
($\vec{B}, \vec{V}$ collinear
$\vec{v} \times \vec{B} = 0$)

(c)



OUT of Paper

(d)

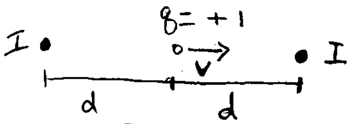


($\vec{B}$ and $\vec{V}$ are each in plan of paper)

Zero Force

$q = 0$

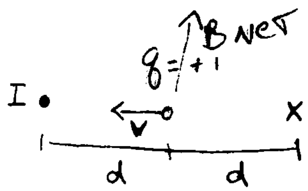
(e)



Zero force

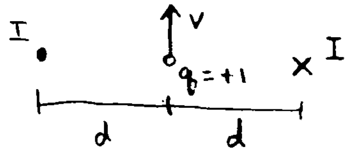
Net $\vec{B} = 0$ at point where charge is

(f)



INTO PAPER

(g)



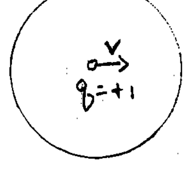
Zero Force

$\vec{B}$ collinear to $\vec{V}$ at point

(h)

$\vec{B}$ out of paper

I (circular distributi

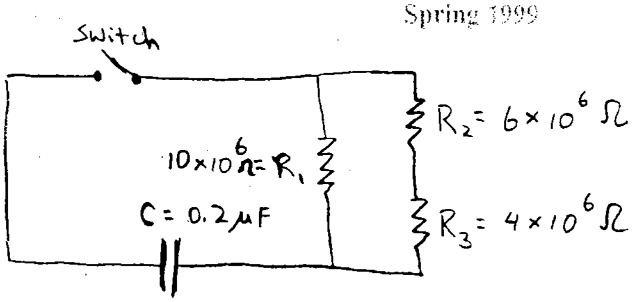


down

NAME S. Manly

Problem 2 (20 pts):

Consider the circuit shown here.



(a) (5 pts) What is the characteristic time constant of this circuit?

$\tau = RC$ Find $R \equiv R_{equivalent}$

$$R_{2\text{-}3} = R_2 + R_3 = 10\ M\Omega$$

$$1/R_{eq} = \frac{1}{R_1} + \frac{1}{R_{2\text{-}3}} = \frac{1}{10\ M\Omega} + \frac{1}{10\ M\Omega} = \frac{2}{10\ M\Omega} = \frac{1}{5\ M\Omega}$$

$R_{eq} = 5\ M\Omega$

$\tau = RC$

$\tau = (5\ M\Omega)(0.2\,\mu$

$\tau = 1$ second

(b) (5 pts) Suppose the capacitor is charged initially with an amount of charge, Q_o, with the switch open. Then the switch is closed. How much charge is there on the capacitor after 10 seconds, relative to Q_o?

$q(t) = q_0 e^{-t/RC}$ or $q : q_0 = e^{-10} : 1$

$$\frac{q(t)}{q_0} = e^{-10/1} = e^{-10}$$

(c) (5 pts) What is the magnitude of the current in the circuit at the moment the switch is closed?

$i(t) = \frac{dq(t)}{dt} = -\frac{q_0}{RC} e^{-t/RC}$ at $t=0$ $|i(0)| = \left|\frac{q_0}{RC}\right| = \frac{q_0}{1}$ Amperes

(d) (5 pts) What is the magnitude of the current in the circuit after 10 seconds, relative to the initial current?

$i(10) = -\frac{q_0}{RC} e^{-10/RC}$ $i(0) : i(10) = i(0) : e^{-10} i(0)$

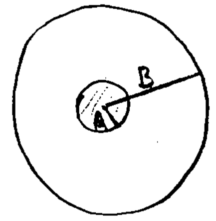
or $1 : e^{-10}$

NAME S. Manly

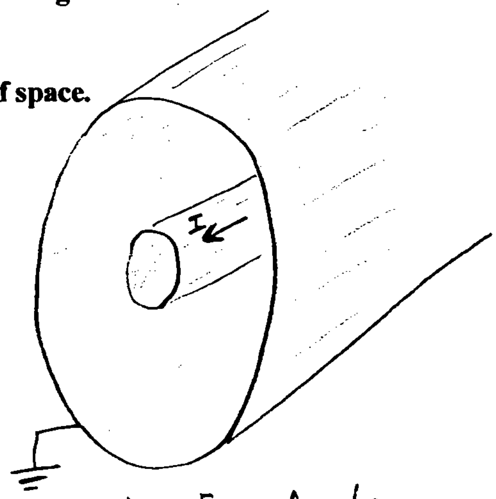
Problem 3 (20 pts):

Consider the system (pictured below), consisting of a thick, infinite, straight conducting wire of radius, A, surrounded concentrically by a thin, infinite, cylindrical conducting shell of radius, B. A current of magnitude I is carried by the internal conductor along the positive Z direction (out of the page, toward you). The current carried by the inner conductor is uniformly distributed as a function of the cross sectional area of the inner conductor. The outer conducting shell carries no current and is grounded. There is no net charge on either conductor.

Use Ampere's Law to find the magnetic field in all regions of space.



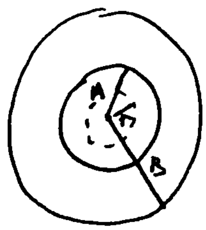
View From Front



View From Angle

Asked to use Ampere's Law.

$$\int_C B \cdot dl = \mu_0 I_{encl}$$



Region I: $r < A$

$$\int B \cdot dl = B \int dl = B\, 2\pi r = \mu_0 \frac{I}{\pi A^2} \pi r^2$$

$$B = \frac{\mu_0}{2\pi} \frac{I r}{A^2}$$

agree at $r = A$ as they should

Region II: $\cancel{A \le r <}$ $A \le r < B$

$$B\, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Region III: $r \ge B$

No change in current enclosed as $r \ge B$

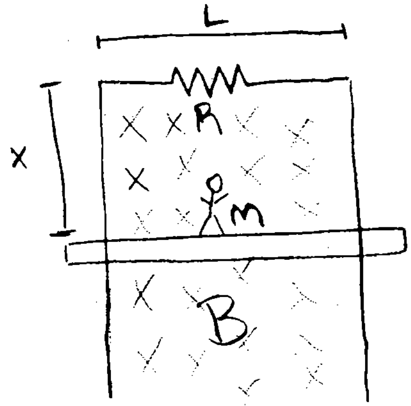
$\therefore$ B unchanged

$$B = \frac{\mu_0 I}{2\pi r}$$

NAME S. Manly

Problem 4 (20 pts):

My friend Chaz is always up to some sort of get-rich-quick scheme. His latest idea is the "magnetic levitation elevator", pictured below. In his design, the elevator is not supported against gravity by cables. Instead he fills the elevator shaft with an adjustable, uniform magnetic field and makes use of the magnetic force on currents induced in the floor of the moving elevator. We can model his system as a horizontal, conducting rod (elevator floor) that can slide up and down two vertical, conducting rods that are connected at the top by a conductor with resistance, R. The sliding rod (elevator floor) is designed to maintain electrical contact with the two vertical poles as it moves up and down. Except for that at the top, there is no other electrical resistance in the system. The sliding rod, elevator floor, and elevator occupants have a combined mass of M. The transverse width of the elevator floor/rod is L.



(a) (4 pts) What is the magnetic flux through the elevator/rod circuit at any given instant?

$$\Phi_B = B x L$$

(b) (6 pts) If the elevator is moving down, what is the magnitude and direction of the current induced in the elevator/rod circuit?

Lenz's law
B flux increasing
i goes to reduce it
$\therefore$ i is counterclockwise

$$\frac{d\Phi_B}{dt} = BL\frac{dx}{dt} = BLv \qquad |\mathcal{E}| = \left|-\frac{d\Phi_B}{dt}\right| = BLv$$

$\mathcal{E} = iR$

$i = \mathcal{E}/R = \frac{BLv}{R}$ magnitude

(c) (10 pts) Suppose the elevator starts from the top floor of the building and begins to descend. The elevator will accelerate downward due to gravity. Find the "terminal velocity" of the elevator, i.e., the final velocity it will have, given the conditions, in terms of L, M, R, and the magnitude of B.

F on induced current is up $= iLB$ up $= \frac{BLv}{R}LB$

$F_{down} = mg$

$F_{NET} = 0$ because at terminal velocity

$$\therefore \quad \frac{B^2L^2v}{R} = Mg$$

$$v = \frac{MgR}{B^2L^2}$$

NAME S. Manly

Problem 5 (16 pts):

You have a friend who recently bought an electrical space heater for his dorm room. The heater is rated at 1000 Watts and is designed to operate on a 240 volt circuit. Unfortunately for your friend, his dorm room is only equipped with 120 volt outlets. For this problem, assume these voltages are direct current (dc) rather than alternating current.

(a) (8 pts) What is the resistance of the space heater?

$$P = \underbrace{i^2 R}_{V = iR} = \frac{V^2}{R} \qquad 1000 \text{ Watts} = \frac{(240 \text{ Volts})^2}{R}$$

$$R = \cancel{\frac{1000}{(240)^2}} \quad \frac{240^2}{1000} = 57.6\,\Omega$$

(b) (8 pts) If your friend goes ahead an plugs it into an outlet in his dorm room, what power output (dissipated as heat) will he get from his heater?

$$P = \frac{V^2}{R} = \frac{(120)^2}{\cancel{240}^2 / \cancel{(240)^2}\ 1000} = \frac{(120)^2\ \cancel{(240)}\ 1000}{(240)^2\ \cancel{1000}} = \cancel{829}\ 250 \text{ watt}$$

NAME S. Manly