P114 University of Rochester NAME Solnberg – SM
S. Manly Spring 2003

Final Exam (May 7, 2003)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

For Problems 1-3 no partial credit will be give. Please use great care on these problems.

Problem 1 (9 pts, 3 parts at 3 pts per part):

An object is located 4 cm from a converging lens of focal length 3 cm. The magnitude of the magnification of the image is

a) 3
b) 4
c) 9
d) 12
e) 16

$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$ $\quad i = 12\text{ cm}$

$\frac{1}{4} + \frac{1}{i} = \frac{1}{3}$ $\quad |m| = \frac{i}{o} = \frac{12}{4} = 3$

The region of space around a moving proton contains

a) a gravitational field only.
b) a magnetic field only.
c) an electric field only.
d) an electric and magnetic field, but no gravitational field.
e) an electric, magnetic and gravitational field.
f) no field of any sort.

A ray of light passes from air into water, striking the surface of the water with an angle of incidence of 45°. Which of the following quantities change as the light enters the water: (1) wavelength, (2) frequency, (3) speed of propagation, and (4) direction of propagation?

a) 1 and 2 only
b) 2, 3, and 4 only
c) 1, 3, and 4 only
d) 3 and 4 only
e) 1, 2, 3, and 4

λ yes
ν NO $\rightarrow$ 1, 3, 4
v yes

Direction – yes because angle of incidence is NOT zero

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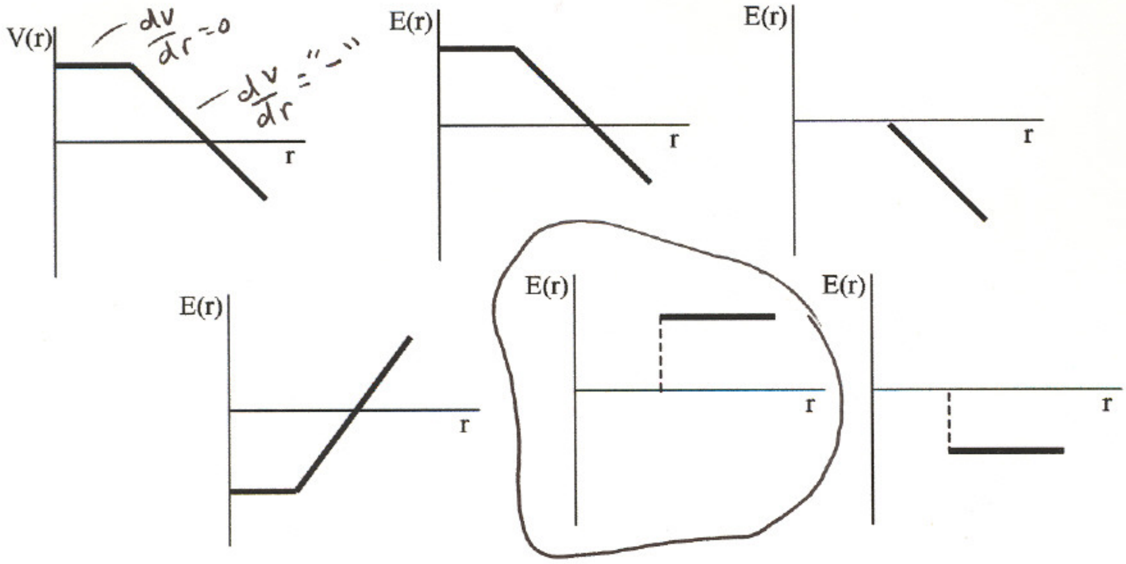
1)	/9
2)	/9
3)	/9
4)	/~~9~~ 10
5)	/~~9~~ 11
6)	/~~9~~ 10
7)	/~~9~~ 10
8)	/~~9~~ 10
9)	/~~9~~ 10
10)	/~~10~~ 11
~~11)~~	~~/10~~
tot	/100

Problem 2 (9 pts, 3 parts at 3 pts per part):

If the potential V of an array of charges is given versus the distance from the center of the charge distribution in the left-most graph, circle the graph that shows the electric field E as a function of distance r.



$E = -\nabla V(r)$

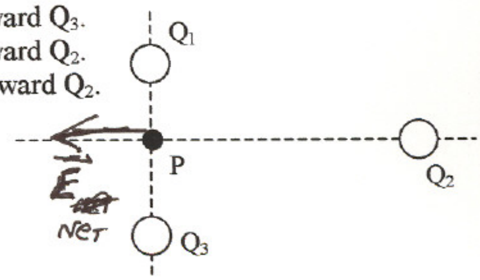
or

$E = -\frac{\partial V}{\partial r} \sim -\frac{dV}{dr}$

Three positive and equal charges Q_1, Q_2, and Q_3 are at the corners of an equilateral triangle as shown. Point P is at the midpoint of the line between Q1 and Q3. The electric field at P is

a) zero.
b) not zero and is directed along the line from P to Q_3, directed toward Q_3.
c) not zero and is directed along the line from P to Q_2, directed toward Q_2.
d) not zero and is directed along the line from Q_1 to Q_2, directed toward Q_2.
e) none of these is correct.

Actually directed away from Q_2 along line from P to Q_2.

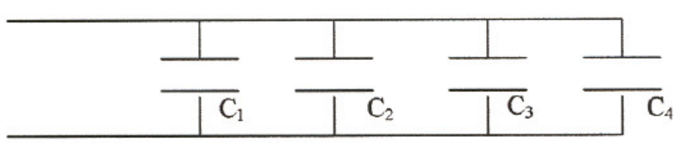


If $C_1 < C_2 < C_3 < C_4$ for the combination of capacitors shown, the equivalent capacitance

a) is less than C_1.
b) is more than C_4.
c) is between C_2 and C_3.
d) is less than C_2.
e) could be any value depending on the applied voltage.

Capacitors in //

$C_{eq} = C_1 + C_2 + C_3 + C_4$



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Problem 3 (9 pts, 3 parts at 3 pts per part):

Biff Jones passes you on a train moving very fast. In fact that train passes you at a constant speed of 0.9c. As Biff passes by, you see that he has a mass oscillating on a spring in the seat in front of him. You measure the period of oscillation of that mass-spring system to be 3 seconds. What period of oscillation does Biff measure for the same system?

a) 3 seconds
b) 2.7 seconds
c) 6.9 seconds
d) 1.3 seconds
e) 3.33 seconds

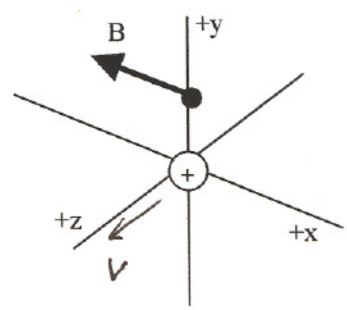
Time period is shortest in proper frame. ← Time Dilation
In this case the proper frame for timing the pendulum Motion is Biff's frame (Moving with the pendulum)

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-.9^2}} = 2.3$$

$$\underbrace{\Delta t'}_{\text{you}} = \gamma \underbrace{\Delta t}_{\text{Biff}}$$

At the instant the positively charged body is at the origin, the magnetic field at point P due to the motion of this charged body is in the negative x direction. The charged body must be moving

a) in the negative z direction.
b) in the potitive y direction.
c) in the positive x direction.
d) in the negative y direction.
e) in the positive z direction.



FROM
Right hand rule
For $\vec{B}$ from Moving charge

Arrange the following types of electromagnetic radiation in order of increasing wavelength: infrared light, gamma rays, ultraviolet light, visible light

a) Gamma rays are not electromagnetic radiation.
b) Gamma rays, infrared, visible, ultraviolet
c) Gamma rays, ultraviolet, visible, infrared
d) Visible, ultraviolet infrared, gamma rays
e) Infrared, visible, ultraviolet, gamma rays

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Problem 4 (9 pts):

The famous hip-hop artist, Vanilla Spongebob, decides to impress audiences on the promotional tour for his new album "Vanilla Shakes". As part of the show, he deposits 0.5 Coulombs of negative charge on his body (using his new Van de Graff-o-matic from Wal*Mart). Then he enters a region where the floor and ceiling (5 m above the floor) are charged and act like a large parallel plate capacitor The electric field between the floor and the ceiling is directed such that it produces an upward force on Vanilla's charged body.

a) What is the potential difference between the floor and ceiling such that Vanilla's body can float stationary in the air above the floor? Assume Vanilla and his costume, including his heavy, bejeweled navel ring, has a mass of 100 kg.

$\uparrow F = qE$

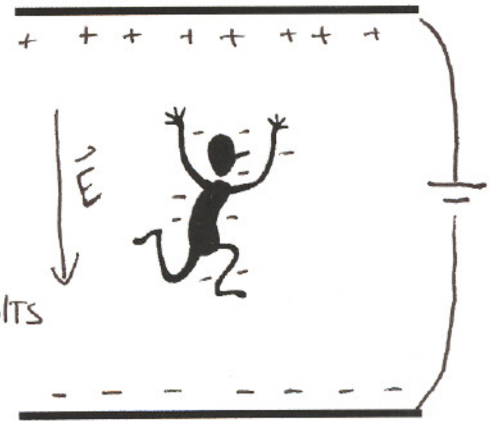
$Q \downarrow mg$

$$(100\,kg)(9.8\,m/s^2) = (0.5\,C)\,E$$

$$E = 1960\ \text{Volts}/m$$

$$E = \frac{\Delta V}{d} = \frac{\Delta V}{5m}$$

$$\therefore\ \Delta V = Ed = (1960)(5) = 9800\ \text{Volts}$$



b) What is the direction of the electric field? Indicate it on the drawing.

Down

c) Assuming the charged plates on the floor and ceiling are 5 m on each side, how much energy does it take for Vanilla to charge the plates for his stunt?

Neglect losses to resistance and any inefficiencies that might exist in his system.

$$u_E = \frac{\epsilon_0 E^2}{2} = \frac{8.85\times10^{-12}}{2}(1960)^2 = 1.7\times10^{-5}\ J/m^3$$

$$\epsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}$$

Total Energy to establish field in volume $= (u_E)(\text{Total volume})$

$$= (1.7\times10^{-5})(5m)(5m)(5m)$$

$$= 2.1\times10^{-3}\ \text{Joules}$$

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Problem 5 (9 pts.):

A solid spherical conductor of radius 15 cm has a charge Q=6.5 nC on it. A second, initially uncharged, spherical conductor of radius 10 cm is moved toward the first until they touch and is then moved far away from it. How much charge is there on the second sphere after the two spheres have been separated?

When the conductors touch they are at the same Potential. The charge will divide between the conducting spheres according to their radii.

$V_1 = \frac{kQ_1}{R_1}$ $\quad V_2 = \frac{kQ_2}{R_2}$ $\quad \therefore \frac{Q_1}{R_1} = \frac{Q_2}{R_2}$ or $\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$

Now $6.5\text{ nC} = Q_1 + Q_2$

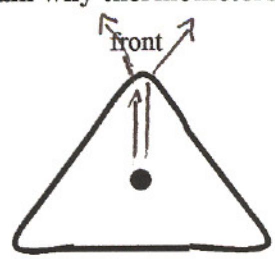
$$6.5\text{ nC} = Q_2 \frac{R_1}{R_2} + Q_2 = Q_2\left(1+\frac{R_1}{R_2}\right) = Q_2\left(1+\frac{15}{10}\right) = Q_2\, 2.5$$

$(Q_2 = 2.6\text{ nC}$ on sphere w/ $R = 10\text{ cm})$ ← 2ND sphere

$6.5 - 2.6 = 3.9\text{ nC}$ on sphere w/ $R = 15\text{ cm}$

Problem 6 (9 points):

Below is a sketch of the cross section of an ordinary glass thermometer for use in a doctor's office. In the middle is a small tube containing a thin "stream" of mercury (or some other substance) that expands with increasing temperature. Surrounding that small tube is glass with a triangular cross section, rounded at the corners. Briefly explain why thermometers are designed with the curved surface on the front.



The rays of light coming from the small tube of mercury are refracted at the curved surface making the small tube look much wider/larger. Thus, it is easier to see.

Problem 7 (9 pts):

Judy Goodall decided to study bats instead of monkeys, much to the chagrin of her famous sister Jane. During a recent field study she made a number of observations that puzzled her. Knowing you were a physics star in college, Judy comes to you and asks you to help explain her observations.

Judy observed the following things about a particular species of bats:

- **This species eats insects.**
- **They "see" by emitting and detecting sound waves at a frequency of 170 kHz.**
- **They only eat insects larger than 2 mm in size.**
- **The speed of sound in air is 343 m/s.**

Please explain briefly, using the physics principles we have discussed recently in this course, why insects smaller than 2 mm are safe from being consumed by the bats.

$v = \lambda \nu$

$\lambda = \frac{v}{\nu} = \frac{343 \text{ m/s}}{170000 \frac{1}{s}} = 2 \text{ mm}$

The wavelength of sound used by the bats is approximately 2 mm. When these waves are reflected off something ~2 mm or smaller the wave is diffracted and the object cannot be resolved or imaged using waves with 2 mm wavelength. The bat can't chase and eat something it cannot see clearly

Problem 8 (9 pts):

A defibrillator sends a 6 A current through the chest of a patient by applying a 10,000 volt potential between two paddles placed on the patient's torso.

a) What is the resistance of the current path through the patient? $V = iR$ $\frac{10000}{6} = R = 1666\ \Omega$

b) What is the energy expended through the defibrillator if the voltage is applied for 0.01 seconds? (Assume the resistance and current are constant throughout the time the voltage is applied.) $P = i^2 R = 6^2\, 1666 = \cancel{5.9}\ 6 \times 10^4 \text{ J/s}$ $P \Delta t = \text{Energy}$

$(6 \times 10^4 \text{ J/s})(0.01) = 600 \text{ Joules}$

c) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. What might you expect to happen if the gel was not used and a larger voltage was used instead in order to generate the same current through the patient?

The Power input to the region of high resistance (Region of contact between the paddles and skin) would be much higher. It is proportional to the resistance. This excess power is dissipated as heat and can cause severe burns on the patient in the areas ~~of the padd~~ where the paddles contact the skin.

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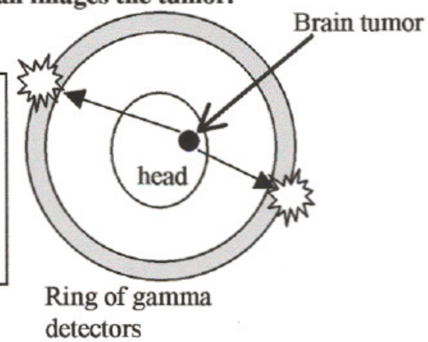
Problem 9 (9 pts):

The medical imaging technique of positron emission tomography (PET) begins by injecting β^+-emitting radiopharmaceuticals into a patient. The β^+ particle is a positively charged electron called the positron. It is the electron's anti-particle. It will travel in the body until it approaches an electron, at which point the positron and the electron will annihilate each other and turn into two 0.511 MeV gamma rays emitted back-to-back. Generally, the positron travels a very short distance before it annihilates with an electron. So the gamma rays are emitted in roughly the location of the initial nuclear decay leading to the positron. PET works by setting up gamma ray detectors in a circle around the body. By detecting the difference in time of detection of the two back-to-back gamma rays, one can determine the position of the annihilation along the flight path of the two gammas. Integrated over many decays in many directions, a three-dimensional image can be formed of the region where the β-decays are taking place. The medical utility of this imaging technique comes about when the nucleus undergoing β-decay is part of an atom that is incorporated into a molecule that tends to concentrate in inflamed regions of the body. The molecule concentrates around the tumor. Thus the decays happen around the tumor and a PET scan images the tumor.

Gamma to right hits before gamma to the left, telling the observer that the distance to the right is shorter than the distance to the left. The difference in time is used to calculate the "exact" position of the positron emission (the tumor).



2 PTS

a) Suppose the isotope $^{13}_{7}N$ is used as the decaying nucleus for a PET scan. What is the electron shell configuration for atomic $^{13}_{7}N$?

$Z = 7 \quad \therefore 7e^- \qquad 1s^2 2s^2 2p^3$

3 PTS

b) Write the relevant nuclear decay equation for the β-decay that takes place.

$$^{13}_{7}N \rightarrow \beta^+ + {}^{13}_{6}X + \nu_e$$

$^{13}_{6}C$

X corresponds to Z = 6 ∴ carbon

only 1 PT for missing the ν

No PTS off if they don't know X is carbon so long as they know it is a different element from N

4 PTS

c) ^{13}N has a half-life of 10 minutes. At noon today, the PET detectors register 1000 counts/minute above background due to the ^{13}N injected into a patient. How many counts/minute will the PET detectors register ~~tomorrow at noon~~ in one hour for the same patient? (For this calculation, please ignore any biological activity that might change the counts from day to day. (*)

$$a(t) = a_0 e^{-\lambda t} \qquad t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \quad \text{or} \quad \lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{10\ \text{min}} = 0.069\ \text{min}^{-1}$$

$$a(t = \cancel{1440}\ 60) = (1000)\, e^{-0.069(\cancel{1440}\ 60)} \cong 16\ \text{counts/min}$$

Problem 10 (11 pts):

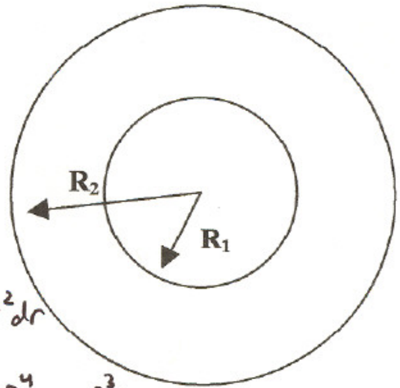
Consider the spherically symmetric charge distribution below. Determine the electric field in the three regions $0<r<R_1$, $R_1<r<R_2$, $r>R_2$. Show your work/reasoning.

For $r<R_1$: Total charge -Q distributed as $\rho(r)=\rho_0(\frac{r}{R_1}-1)$

For $R_1<r<R_2$: $\rho(r)=0$

At $r=R_2$: Conducting shell with a total charge of +Q

For $r>R_2$: $\rho(r)=0$



$\underline{r<R_1}$ $\oint \vec{E}\cdot d\vec{A} = E4\pi r^2 = \frac{\int \rho dV}{\epsilon_0} = \frac{\int \rho 4\pi r^2 dr}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \rho_0(\frac{r}{R_1}-1)4\pi r^2 dr$

$= \frac{4\pi\rho_0}{\epsilon_0}\left[\int_0^r \frac{r^3}{R_1}dr - \int_0^r r^2 dr\right] = \frac{4\pi\rho_0}{\epsilon_0}\left[\frac{r'^4}{4R_1}\Big|_0^r - \frac{r'^3}{3}\Big|_0^r\right] = \frac{4\pi\rho_0}{\epsilon_0}\left(\frac{r^4}{4R_1}-\frac{r^3}{3}\right)$

$$E = \frac{\rho_0}{\epsilon_0}\left(\frac{r^2}{4R_1}-\frac{r}{3}\right)\hat{r}$$

$\underline{R_1<r<R_2}$ $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = \frac{-Q}{\epsilon_0}$ $E4\pi r^2 = -\frac{Q}{\epsilon_0}$ $\boxed{\vec{E} = -\frac{Q}{\epsilon_0 4\pi r^2}}$

$\underline{r>R_2}$ $\oint E\cdot dA = \frac{Q_{encl}}{\epsilon_0}$ $Q_{encl} = +Q + (-Q) = 0$

$\vec{E} = 0$ for $r>R_2$

Whew! Welcome to the end of your introductory physics sequence! I hope you will find some of what you have learned about useful through the years. You've been great sports.

As for grades: I must make a research trip from Thursday-Sunday. During that time the finals will be graded. On Monday, May 12 I will try to process and turn in the P114 grades. You should be able to see your grade on Access shortly after I turn them in. On May 13, I need to do the same for P121. After that, I will go through a rather elaborate procedure to send a report via email to each of you that contains the numbers in my spreadsheet that were used to calculate your grade. You can check those for errors and contact me provided any error you discover would change your letter grade (information for that determination will be provided). All inquiries concerning grades will be ignored until I have processed the grades and sent the spreadsheet numbers to you. You can pick up your graded final exams once I turn your grades in to the registrar (by the evening of the 12th hopefully). Trust me. I am more anxious to get your grades out than you are to get them!

Have a wonderful summer! Good luck moving out into the real world if you are graduating! -- SLM