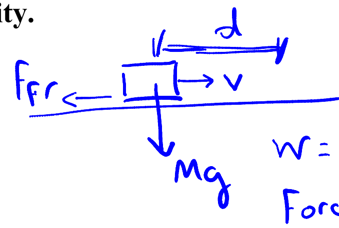
NAME Soln key - Slm

Final Exam (December 18, 2006)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (5 pts, justify your answer):

A body of mass M slides a distance d along a horizontal surface with a coefficient of kinetic friction μ_s. Determine how much work is done by gravity.



a) mgd
b) zero
c) –mgd
d) One cannot tell from the information provided.
e) None of these is correct.

$W = F \cdot d = 0$

Force of gravity $\perp$ to displacement vector

Problem 2 (5 pts, justify your answer):

A golf ball and a ping-pong ball are dropped from the same height in a vacuum chamber. When they have fallen halfway to the floor, they have the same

a) speed.
b) potential energy.
c) kinetic energy.
d) momentum.
e) speed, potential energy, kinetic energy, momentum.

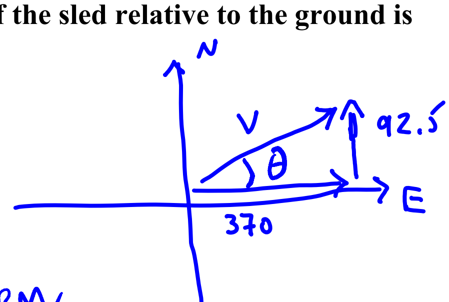
For both objects $a = g$
So speeds will be the same at same point in motion

$PE \sim mgh$
$KE \sim \frac{1}{2} mv^2$
$P \sim mv$
all depend on m and will differ

Problem 3 (7 pts, justify your answer):

Santa and his new turbo-charged sled are heading due east. Santa's airspeed indicator shows that the sled is moving at a speed of 370 km/h relative to the air. If the wind is blowing from the south at 92.5 km/h, the velocity of the sled relative to the ground is

a) 357 km/h at 14 degrees east of north
b) ~~381 km/h at 104 degrees east of north~~
c) 381 km/h at 76 degrees east of north
d) 357 km/h at 76 degrees east of north
e) 381 km/h at 14 degrees east of north



$V^2 = 92.5^2 + 370^2$ $V = 381$ km/h

$\tan\theta = \frac{92.5}{370}$

$\theta = 14°$ $90 - 14 = 76°$ East of North

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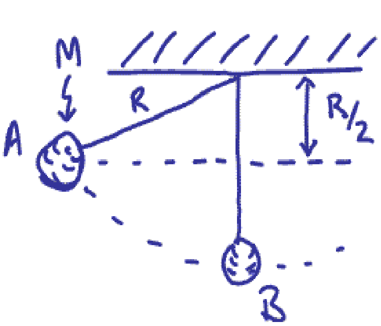
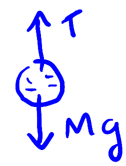
Problem 4 (8 pts, show your work):

A mass M of 0.1 kg attached to a string of length R=1.0 m is released from rest at point A. Determine the tension in the string as it passes through the lowest point of its motion B.

1)	5 /5
2)	5 /5
3)	7 /7
4)	8 /8
5)	8 /8
6)	7 /7
7)	10 /10
8)	10 /10
9)	10 /10
10)	10 /10
11)	10 /10
12)	10 /10
tot	100 /100



FBD at pt B



Mass M moves on circular path

So $\Sigma F_y = m\frac{v^2}{R} = T - Mg$

$T = \frac{mv^2}{R} + Mg$

Use Energy conservation to determine V at point B

$Mgh = \frac{1}{2}Mv^2$

$Mg\frac{R}{2} = \frac{1}{2}Mv^2 \Rightarrow v^2 = gR$

$T = Mg + Mg$

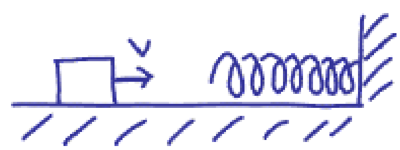
$\boxed{T = 2Mg} = 2(0.1)(9.8)$

$= 1.96$ Newtons

Problem 5 (8 pts, show your work):

As shown in the sketch below, 5 kg box slides into a spring of spring constant 310 N/m compressing it 24 cm.

a) **(3 pts) What is the incoming speed of the block?**



From Energy Conservation KE of box $\longrightarrow$ PE of compressed spring

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \qquad v^2 = \frac{k}{m}x^2 = \frac{(310\ \frac{N}{m})(.24\ m)^2}{5\ kg} = 3.6$

$\boxed{v = 1.9\ m/s}$

unit check ✓ $\frac{kg\ m/s^2\ m}{kg} \rightarrow m^2/s^2$

b) **(5 pts) How long is the box in contact with the spring before it bounces off in the opposite direction?**

During time the box is in contact w/ spring, system executes SHM

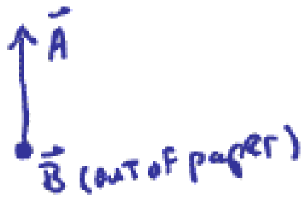
Box comes off spring as spring pushes box back through point of equilibrium length of spring (midpoint of SHM) because at this point spring decelerates and box does not. So box in contact w/ spring for ½ period

$\omega^2 = k/M \qquad \omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{5}{310}} = 0.8\ s$

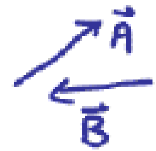
$\boxed{t_{contact} = \frac{T}{2} = 0.4\ s}$

Problem 6 (7 pts, no need to justify, be careful, within each part no partial credit will be given):

For each part below, indicate the direction of the specified quantity referring to the sketch. Your choices are left, right, up, down, into the paper, out of the paper and no direction (either not a vector or has a magnitude of zero).



(a) $\vec{A} \times \vec{B}$ To Right



(b) $\vec{A} \times \vec{B}$ out of paper

Disk spins clockwise Abt Axis

(c) $\vec{\omega}$ into paper

Disk spins clockwise Abt Axis
Pushed up against surface that exerts a frictional force

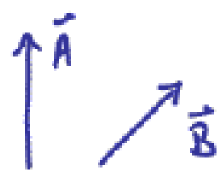
(d) $\vec{\omega}$ into paper

Disk spins clockwise Abt Axis
Pushed up against surface that exerts a frictional force

(e) $\vec{\alpha}$ out of paper

Disk spins clockwise Abt Axis
Pushed up against surface that exerts a frictional force

(f) $\vec{L}$ into paper



(g) $\vec{A} \cdot \vec{B}$ No Direction Not a vector

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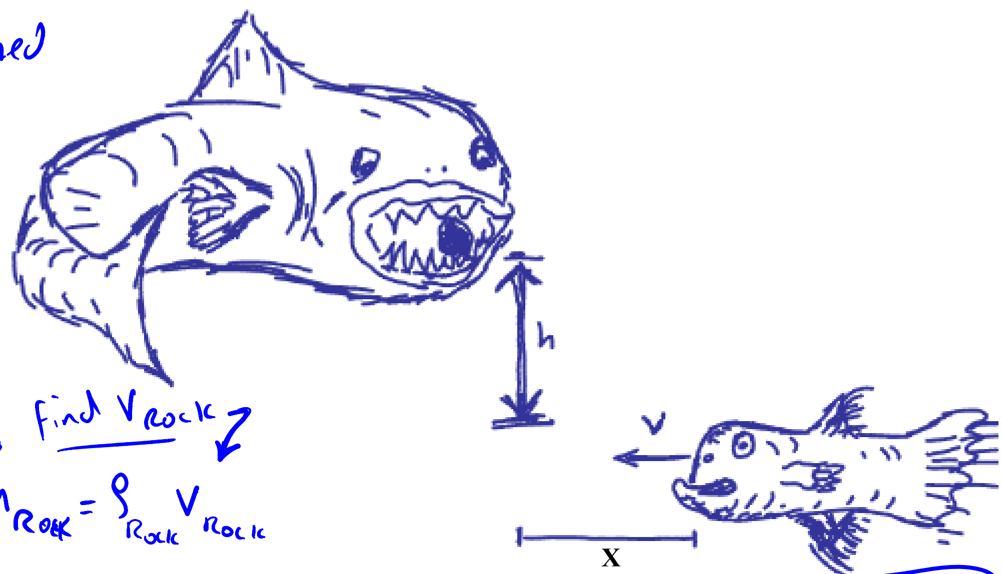
Problem 7 (10 pts, show your work):

Sammie the Fish, a Gambino crime family thug operating in the waters off the Port of New York, has instructions to eliminate Slow Joe the Carp who is half-brother of Ariel the Little Mermaid. Sammie plans to carry out his evil deed by dropping a rock on Slow Joe as Slow Joe swims beneath him. When the time comes, Sammie spots Slow Joe swimming in a straight horizontal line at a speed of 1 m/s and sets his trap. Sammie picks up a 3 kg rock (with specific gravity of 2.4) in his teeth and swims to a height (h) of 10 m above the level where Slow Joe is swimming. If Sammie is to drop his rock and hit Slow Joe, how far is slow Joe horizontally from Sammie (how large is x in the sketch below) when Sammie releases his rock? (Assume the seawater has a specific gravity of 1. Assume there is no viscosity or friction in this problem.)

FBD of rock once it is Dropped

$\uparrow F_B$ = Bouyant Force

$\downarrow mg$



$ma = F_B - mg$

F_B = weight of displaced water $= \rho_W V_{Rock}\, g$

Find V_{Rock}

$M_{Rock} = \rho_{Rock} V_{Rock}$

$\frac{M_R}{\rho_W} = \frac{\rho_{Rock}}{\rho_W} V_{Rock}$

$\frac{\rho_{Rock}}{\rho_W} = 2.4 = Sg_{Rock}$ spec. grav of Rock

$F_B = \rho_W \frac{M_R}{\rho_W} \frac{1}{Sg_{Rock}}\, g$

$F_B = M_R \frac{1}{Sg_{Rock}}\, g$

$F_B = (3\text{ kg})\left(\frac{1}{2.4}\right)(9.8\text{ m/s}^2)$

F_B = 12.2 Newtons

Now

$ma = F_B - mg$

$|a| = \left|\frac{12.2}{3} - 9.8\right| = |-5.7|$

$a = 5.7\text{ m/s}^2$ down

$h \downarrow$ drops w/ const a for a dist h in time t

$\leftarrow x$ Moves w/ const v a dist x in t

Slow Joe: $x = vt$ — ①

Rock: $y = y_0 + v_{0y}t + \frac{1}{2}at^2$

$h = \frac{1}{2}at^2$ — ②

Solve ① and ② Simultaneously eliminate t Solve for x

h, a both negative... signs cancel

$h = \frac{1}{2}a\frac{x^2}{v^2}$

$x = \sqrt{\frac{2h}{a}v^2} = \sqrt{\frac{(2)(10)}{5.7}1^2} = 1.9\text{ m}$

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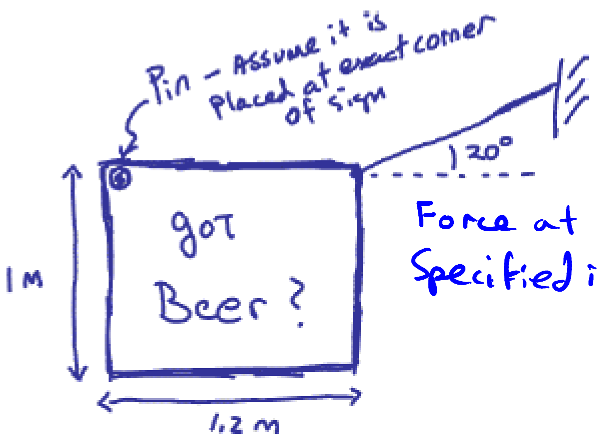
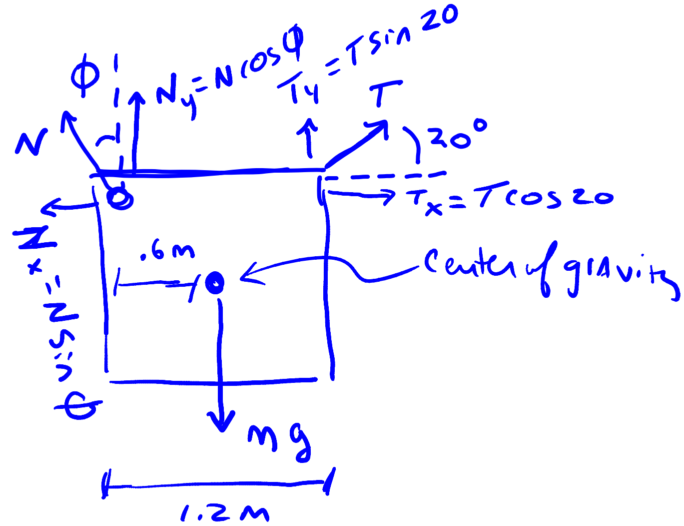
Problem 8 (10 pts, show your work):

Your buddies in the fraternity next door ask for your help in putting up a sign in front of their house. The sign is rectangular (1.2 meters x 1.0 meter) and is supported at one corner by a pin (about which the sign is free to rotate) and in the other corner by a cable that makes an angle of 20 degrees with the horizontal. The basic configuration is shown in the sketch below. The sign has a mass of 5 kg that is distributed uniformly. The system is in equilibrium. Assume the pin is at the exact corner of the sign for simplicity. Determine the direction and magnitude of the force on the pin at equilibrium and the tension T in the cable.



Force at pin not specified in problem



Equilibrium → $\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma \tau = 0 \end{cases}$

① $\Sigma F_x = T\cos\theta - N\sin\phi = 0$

② $\Sigma F_y = N\cos\phi + T\sin\theta - Mg = 0$

$\Sigma \tau = 0$ Evaluate abt pin position (+) τ would give clockwise α

③ $.6\,Mg - 1.2\,T\sin\theta = 0$

$$\boxed{T = \frac{.6\,Mg}{1.2\sin\theta} = 72 \text{ Newtons}}$$

① → $N\sin\phi = T\cos\theta = 72\cos 20 = 68$ Newtons

② $N\cos\phi = Mg - T\sin\theta$
$= (5)(9.8) - (72)\sin 20 = 24$ Newtons
$\quad 49 - 25$

$\frac{①}{②} = \tan\phi = \frac{68}{24} = 2.83$

$\boxed{\phi = 71°}$

use ①

$$\boxed{N = \frac{68}{\sin 71} = 72 \text{ Newtons}}$$

I have rounded these answers ∴ student's answer may vary slightly

A student might also arrive at these answers using equilibrium and symmetry arguments

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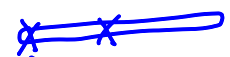
Problem 9 (10 pts, show your work):

A uniform, thin rod of length L=0.5 m and mass M rests on a frictionless horizontal surface. The rod is pivoted about a fixed frictionless axis at one end. The rod is initially at rest. A bullet traveling parallel to the horizontal surface and perpendicular to the rod with speed v = 300 m/s strikes the rod at its center and becomes embedded in it. The mass of the bullet is one-sixth that of the rod.

a) **(6 pts) What is the angular velocity of the rod after the bullet has become embedded?**

KE NOT conserved since bullet embedding is NOT an elastic collision.

$I = \frac{1}{12} ML^2$ → middle Axis



$L_{init} = L_{final}$ determine I_{TOT}

$\frac{L}{2} m_{bull} v_{bull} = I_{TOT}\,\omega$

$I_{TOT} = I_{rod} + I_{bull}$

$I_{TOT} = I_{rod} + M_{bull}\left(\frac{L}{2}\right)^2$

$I_{rod} = I_{rod\,middle} + M\left(\frac{L}{2}\right)^2$ ← ∥ Axis Theorem

$I_{rod} = \frac{1}{12} ML^2 + M\frac{L^2}{4} = \frac{1}{3} ML^2$

use $M_{bull} = \frac{1}{6} M$

$I_{TOT} = \frac{1}{3} ML^2 + \frac{1}{6} M \frac{L^2}{4} = \frac{3}{8} ML^2$

$\frac{1}{3} + \frac{1}{24} = \frac{9}{24} = \frac{3}{8}$

$\frac{L}{2}\frac{1}{6} M v_{bull} = \frac{3}{8} ML^2 \omega$

$\omega = \frac{1}{12} M L v_{bull} \frac{8}{3} \frac{1}{ML^2}$

$$\omega = \frac{2}{9}\frac{v_{bull}}{L} = \frac{2(300)}{9(0.5)} = 133 \text{ Rad/s}$$

b) **(4 pts) What is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bullet before the collision?**

$$\frac{KE_{syst}}{KE_{bull}} = \frac{\frac{1}{2} I_{TOT}\omega^2}{\frac{1}{2} M_{bull} v_{bull}^2} = \frac{\frac{1}{2}\frac{3}{8} ML^2 \left(\frac{2}{9}\right)^2 \frac{v_{bull}^2}{L^2}}{\frac{1}{2}\frac{1}{6} M v_{bull}^2} = \frac{\frac{1}{2}\frac{3}{8}\frac{4}{81}}{\frac{1}{12}} = \frac{(18)4}{(8)(81)}$$

$$\frac{KE_{syst}}{KE_{bull}} = 0.1$$

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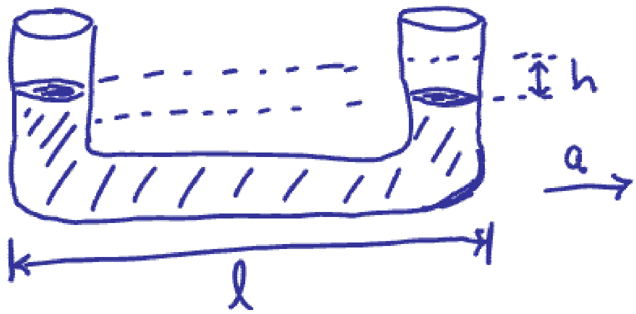
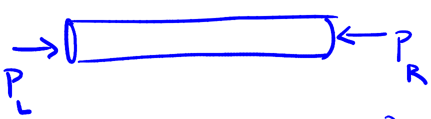
Problem 10 (10 pts, show your work):

A "U-shaped" tube with a bottom horizontal portion of length 0.2 m contains a liquid. What is the difference in height (h) between the liquid columns in the vertical arms if the tube is accelerated toward the right with an acceleration of 2 m/s^2.



Look at force necessary to accelerate fluid in horizontal part of tube



has cross sectional area A

$P_R A = F_R$

$P_L A = F_L$

$$F_L - F_R = M_{Liquid}\, a = \rho_{liquid} \underbrace{\ell A}_{\text{vol of horizontal section of liquid}} a$$

$$P_L A - P_R A = \rho \ell A a$$

$$\underbrace{\rho h g}_{(P_L - P_R)} A = \rho \ell A a$$

$$h = \frac{\ell a}{g} \qquad \frac{m \; m/s^2}{m/s^2} \text{ units ok}$$

$$h = \frac{(0.2\, m)(2\, m/s^2)}{9.8\, m/s^2} = 0.04\, m$$

$$\boxed{h = 4\, cm}$$

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Problem 11 (10 pts, show your work):

Two masses are connected with a massless cable that passes over two massless, frictionless pulleys as shown below. The masses start from rest and are released. The left-hand pulley is attached to the ceiling. The right-hand pulley moves up and down with m_2. Please determine the correct expression for the magnitude of the acceleration of m_1. As shown in the sketch below, the direction of this acceleration is defined to be positive upward. The correct expression is one of the six answers listed below. You must show your work to receive credit for this problem.

(a) $a = g$

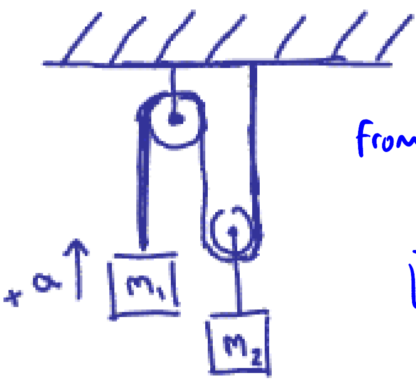
(b) $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$

(c) $a = g/2$

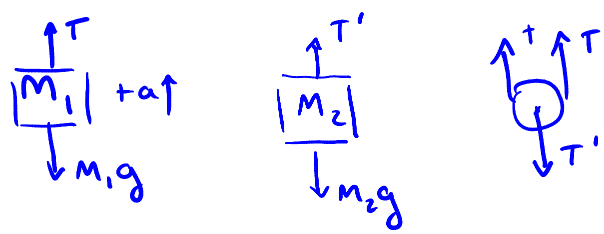
(d) $a = \left(\frac{m_2 - 2m_1}{m_2 + 2m_1}\right) g$

(e) $a = \left(\frac{2m_2 - m_1}{2m_2 + m_1}\right) g$

(f) $a = 2\left(\frac{m_2 - 2m_1}{m_2 + 4m_1}\right) g$



from layout if m_1 moves up w/ a, m_2 moves down w/ $\frac{1}{2}a$



$2T = T'$ (pulley is massless)

$m_1 a = T - m_1 g \rightarrow T = m_1(a+g)$

$m_2\left(\frac{1}{2}a\right) = m_2 g - T' = m_2 g - 2T$

$\frac{m_2 a}{2} = m_2 g - 2m_1(a+g)$

$\frac{m_2}{2} a = m_2 g - 2m_1 a - 2m_1 g$

$\left[\frac{m_2}{2} + 2m_1\right] a = (m_2 - 2m_1) g$

$$\boxed{a = \frac{2g(m_2 - 2m_1)}{m_2 + 4m_1}}$$

Ans. is (f)

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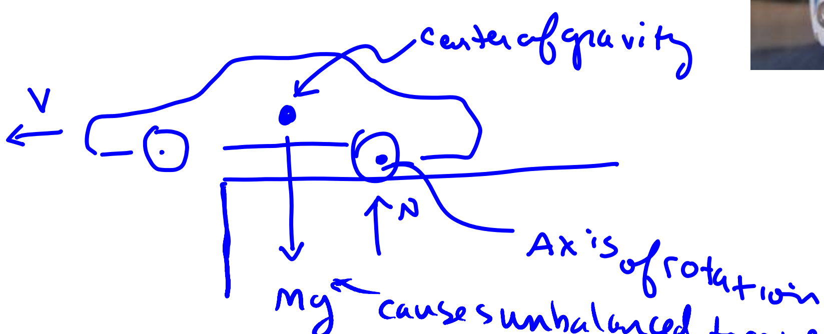
<u>Problem 12 (10 pts):</u>

After graduation you start a Hollywood movie and cuticle consulting business. Ace Frick, the world famous action movie producer, comes to you with a question. He says, "My problem is that every time we send a car horizontally over a cliff, the nose of the car dips down (as shown in the strobed photograph below). You see, in this action scene I want the car to remain horizontal as it falls after being driven off the edge of the cliff at a high speed. Can you tell me why the car does not stay horizontal as it flies through the air? Why is it that the nose dips?"

Below, please provide a brief answer to Ace's question using the physics principles you have learned in Physics 113. Feel free to use equations, sketches or formulas if you wish, but try to make the answer coherent and logical.



The center of gravity of the car is located between the wheels along the length of the car. So, when the front wheels move off the edge of the cliff, there is an unbalanced torque about the back wheels. So $\tau = I\alpha$ means that the car begins to rotate in such a fashion the front end dips down. As the back wheels move off the cliff, there is no longer an unbalanced torque but car continues to rotate with constant ω by Newton's first law.

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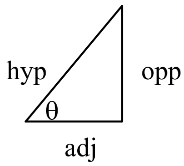
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$



$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$x = x_o + \left(\frac{v_o + v}{2}\right)t$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \int_{t_o}^{t} v\,dt$$

$$v - v_o = \int_{t_o}^{t} a\,dt$$

$$\sum \vec{F} = m\vec{a}$$

$$F_{friction} = \mu_k N$$

$$F_{friction} = \mu_s N$$

$$F_{centripetal} = \frac{mv^2}{r}$$

$$\text{circumference of circle} = 2\pi r$$

$$\text{area of circle} = \pi r^2$$

$$\text{quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \lambda\nu$$

$$T = \frac{1}{f}$$

$$\omega^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\vec{F}_{spring} = -k(\vec{x} - \vec{x}_o)$$

$$work = \int F \cdot ds$$

$$power = \frac{dw}{dt}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$KE_{translation} = \frac{1}{2}MV^2$$

$$KE_{rotation} = \frac{1}{2}I\omega^2$$

$$I = \sum m_i r_i^2 = \int r^2 dm$$

$$X_{cm} = \frac{\sum x_i m_i}{M} = \frac{\int x\,dm}{M}$$

$$I = I_{cm} + mh^2$$

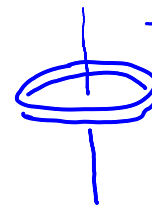
$$\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$P = P_o + \rho g h$$

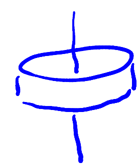
$$AV = \text{constant}$$

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



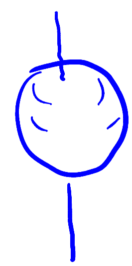
Thin hoop radius R

$I = MR^2$



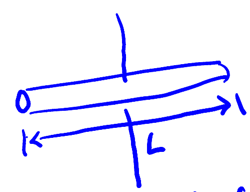
Solid cylinder Radius R

$I = \frac{1}{2}MR^2$



uniform Sphere Radius R

$I = \frac{2}{5}MR^2$



uniform rod of length L

$I = \frac{1}{12}ML^2$